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Fitting a static supply and demand function for labor

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FITTING A STATIC SUPPLY AND DEMAND FUNCTION FOR LABOR

by

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INTRODUCTION

General Nature of the Problem

Wage theories have been put forth for as long a time as economics has been a field of study. The objectives of the various theories have varied considerably. Currently, the nature of the supply of and demand for labor is receiving much attention. Primary interest has centered around the general level of wages (real and monetary), wage determination, and the factors influencing supply of and demand for labor.

An important question of labor supply and demand has been with us ever since Keynes' great work appeared showing that it is possible to have general market equilibrium without full employment. Keynes (23) held that the demand for labor depended upon the real wage, but that the supply of labor depended on the money wage. The crucial question is, "Are the supply of and demand for labor a function of the real or money wage?"

There is little doubt that the demand depends on the real wage, but various reasons have been suggested as to why the supply might depend on the money wage. Included are money illusion, past contracts, and non-rational behavior.

If the supply and demand are a function of the real wage (*i.e.*, the money wage deflated by the price index), we say that the functions are homogeneous of degree zero with respect to the prices and wages. It is possible to test empirically derived functions for homogeneity properties. The question as to whether the supply and demand functions for labor are homogeneous in the prices is important to both economic theory and policy; yet, it has received surprisingly little attention. Tintner (43) has written a theoretical article on homogeneity of the supply of and demand for factors of production, but no empirical work has been done on this aspect in this country. Knowledge of the aspect of homogeneity in supply and demand functions is vital for building good theory and necessary for sound economic policy. This is especially true with respect to labor. The work in this thesis is an attempt to gain more insight into this ambiguous aspect by giving at least a partial and tentative answer to the questions: Are the demand and supply functions of labor homogeneous of degree zero in prices and wages? Do they depend upon real wages rather than money wages?

These questions will be investigated by a time series analysis of the total labor supply in the United States and the labor demand of the bituminous coal industries for the period 1900-1955. The functions are derived by ordinary multiple regression and weighted regression. Because of the important and difficult problems in such an analysis, the work should also be valuable as a study in methodology.

Present Status of Wage Theory

The area of wage theory is conspicuous because of the large number of arm-chair theories that have been put forth. In spite of this large amount of theorizing our knowledge and command of this area are comparatively slight (41). Recently several economists have become highly critical of the present status of wage theory and the procedure and progress being made. Worthy suggestions have been given by Ross (38), Reynolds (37), Myers (30), Kerr (22), and Pierson (34). They agree that we should formulate more realistic models and theories through a greater amount of empirical work.

The major reason that so many of the theories are retained is that little evidence is presented to support or refute any of them. The question to be investigated in this thesis is an example of a definite lack of empirical evidence. It is evident that we are a long way from the point of diminishing returns as far as empirical work in this area is concerned.

A theory of wages must identify the question or questions it seeks to answer. It is not historically correct to speak of wage theory as progressing from inaccurate to more accurate. A theory developed in one period may not be applicable in a later period because of a change in the structure of the economy or a change in the questions that are relevant. Interesting accounts of the history and role of wage theory have been written by Dunlop (14), Woytinsky (53), and Rothschild (40). These men point out the changing role and influence of past theories.

The theories can be grouped into three broad categories: the wage fund theory of the classical period; marginal productivity theories up to the Great Depression; the contemporary period not characterized by any one type of theory. During the classical period, interest centered mainly on the distribution of national product between the social classes and the general level of wages. During the time of Adam Smith, the subsistence level theory was first applied. Wages were expected to tend towards that level which was necessary to support a worker and his family. This theory has been aptly described as the iron law of wages. If the wage tended to rise above the subsistence level, the population would increase and the wage would fall. If the wage fell below the stipulated level, starvation would result, and wages would eventually rise.

The foregoing theories tended to explain the wage level entirely from the supply side. At the beginning of the nineteenth century, the wage fund theory evolved. Under this theory, the wage level is determined by the magnitude of the laboring class and a certain portion of a nation's capital. It was stipulated that a certain fixed amount of capital was relegated to the payment of labor, and the level would thus be determined by the number of laborers among which this fund had to be divided. The theory broke down on the assumption of a fixed wage fund.

The marginal productivity theories dealt with the following questions: "How is the product distributed among the factors that produce it?" "What is the marginal productivity of labor?" "How is the rising productivity of industry distributed?"

The forces operating in these theories include the employer striving to maximize his profits and the laborer trying to maximize his utility (21). The demand schedule was a fairly logical and simple deduction, but explanation of the supply schedule remained inadequate. Explanation was attempted in terms of the pain or disutility of work in relation to the wage, but the concepts were for the most part non-operational. Explanation of labor supply still remains inadequate.

Contemporary theories started with the impetus from the great works of Keynes (23) and Douglas (12). Douglas sought an explanation of occupational and geographical wage differentials and the elasticities of supply and demand for labor. Keynes was interested in the homogeneity property of supply as an essential element in his proof of general equilibrium at less than full employment. More empirical research is needed to determine whether Keynes' hypothesis is valid. It is difficult to give a concise description or even a list of the relevant questions that exist in the area of wage theory. It is certain that the questions are too diverse to be answered by any one theory.

It is interesting to note that Douglas had only two major questions in mind when he set out to derive empirical functions. The results of his work were encouraging and illuminating, yet very little similar work has been done since then.

The use of intuition and pure deductive reason is not adequate. It is necessary to supplement these methods with extensive empirical work. Wage theory is an area where a bridge between theory and the real world is greatly needed. Econometric tools may prove helpful in this respect.

Econometric Methodology

Econometrics, mathematical economics, and statistical economics are particular approaches in the general field of economic science. They are being developed because of a felt need for concrete formulations and measurement and for more intensive empirical work. For purposes of definition we can label statistical economics as the collection and summarizing of economic data and mathematical economics as the formulation and expressing of economic theory in mathematical form.

Econometrics as described by Tintner (47) is a combination of these two methods in an attempt to find numerical values for postulated economic laws and regularities.

In the theoretical stage many economists employ mathematics for deriving economic relationships starting from *a priori* assumptions and ideas about maximization (46). The use of the above methods does not in any way minimize the importance of economic theory. Good theory formulated in a way which permits measurement is a prerequisite for application of econometrics.

Econometrics does not insure against contradictory assumptions, mistakes made by the econometrician, or poor judgment. Used in a proper fashion and under an awareness of limitations, it does lead toward the more precise formulation and measurement needed for the discipline of economics. To those who claim that human action cannot be reduced to mathematical formula, we have no answer as yet. The value of econometrics must be judged by results from its employment.

Application of Econometrics in Wage Theory

In wage theory there is first, a need for more concrete formulation of questions, and second, a need for more precise measurement of relationships (elasticities of supply and demand, productivity, etc.). With the exception of the work of Douglas there has been very little application of econometrics in this area. Undoubtedly, one of the major reasons is the extreme complexity of this area of economics.

Another reason might be due to the skepticism shown by Keynes toward econometrics. Keynes did not employ mathematics in the formulation of his theory, nor did he apply statistical tools in empirical analysis. Since his time various other men have expressed and extended his theories in mathematical terms but they have not attempted to verify them (27).

Keynes (24) stated that he preferred the mazes of logic to the mazes of arithmetic. He used the phrase, "mazes of arithmetic", as a name for Tinbergen's econometric and statistical analysis in economics. Keynes was skeptical of the possibility of ever adapting statistical techniques to the conditions in economic research.

It is unfortunate that only a few results exist in this country for judging the application of econometrics in wage theory. Douglas (11), who is responsible for the most extensive econometric analysis in this area, believes that it is possible to derive empirical relationships in economics with econometric analysis.

It is necessary to be aware of the large number of psychological and other difficult to observe factors that will be present in any question in this area. To labor continually over the complexity of the problems is of no avail, however. Until more actual investigations are made, we will not know the true extent of the difficulties, nor will we be able to develop means of handling them.

We hold no particular claim that econometricians or even any economist will have the background and training to answer best the questions that arise. The most important aspects may be in non economic sectors (38). For interesting accounts of the scope and philosophy of economics the reader is better referred to articles by Lange (25) and Collingwood (8).

Many of the interesting questions can be expressed in a form or model in which it is possible to test them against appropriate data. Also, there are several statistical tools for analyzing data that have proved reasonably successful. As to further discussion of the applicability of econometrics, it is better to do so in relation to a specific question.

EXPRESSING THE PROBLEM IN SUITABLE FORM FOR ECONOMETRIC ANALYSIS

The optimum working arrangement in research evolves from the integration of knowledge in three areas: knowledge of theory, knowledge of the capabilities and limitations of various techniques of analysis, and the greatest possible knowledge of the subject area. There is no one method of analysis that will be applicable to all questions, nor does it follow that there is always some econometric tool that will be adequate.

Some questions do not lend themselves to expression in quantitative terms. The political aspects of unions raised by Ross (38) do not readily adapt to econometric analysis; however, with further improvement in the application of game theory (33) there is real possibility of adequately handling such factors. It should not be contended that such aspects as political and institutional factors will always defy quantitative expression. We hope that the measurement of more objective characteristics can give a fairly good measure of more subjective qualities. On the other hand, it should not be contended that our knowledge is nil if quantitative terms are lacking. Interesting suggestions were presented at the University of Chicago symposium (26) on the topic of "Quantification: the quest for precision". The conclusion was reached that quantification adds much to knowledge but it does not insure complete knowledge.

The main criticism of past work in wage theory is that most economists have not formulated hypotheses and questions in a form permitting measurement and testing. Consequently, there have been few attempts to verify or reject the numerous theories. This chapter is devoted to suggestions on this relevant aspect.

Investigating a Specific Problem

There is little hope of developing a wage theory that will make it possible to deduce answers to all pertinent questions. The present literature demonstrates the complexity and diversity of problem areas. There are many questions to be answered; the questions are often complex and pertain to a wide variety of conditions. There is also a decided lack of empirical knowledge, plus a lack of methodology, adequate to cope with the problems that arise in empirical investigations. For these reasons it is best to work on a specific problem, designing and modifying models to suit best the specific instance at hand.

In the classical period it was more realistic to speak in terms of one theory because economists were essentially interested in only one question - what relative share of national product was going to labor? Now, there are a great number of implications in the labor market that are of primary interest to both economic policy and theoretical economics. We will, naturally, strive for as unified a theory as possible; but to completely disregard the availability and limitations of methods and data would be unwise. The abstractions necessary for relatively simple models must be understood and adequately discussed in each study. Many questions are important enough to warrant separate investigation, and any reasonable results will be worth the expense.

In an empirical science such as economics the terms used and the hypothesis put forth should be capable of empirical interpretation. This is not easily done because the large number of relationships that exist requires a certain degree of abstraction to make the procedures operational. The basic idea of operationalism is the demand that the concepts used in the description of experience be framed in terms of operations which can be unequivocally performed. Caution must be exercised in the use of operational philosophy for it is possible to put too much emphasis on operationalism at the expense of adequate theoretical import. It cannot be said that there is no knowledge unless there is operationalism, but the present status of wage theory certainly indicates a great need for such in the form of testing and verifying theories. Intensive empirical investigations are needed. Hempel (20) and Benjamin (2) have emphasized the advantages and limitations of operationalism in various subject areas.

In the social sciences it is usually not possible to isolate the specific relationship in which we are interested from other relationships. There is small opportunity for experimentation as in the natural sciences. This leaves only the possibility of conceptual isolation of a particular relationship (7). Consequently, it is necessary to estimate the effects of other relationships. This is an important aspect in evaluating the results of empirical studies. The degree to which it is best to isolate a particular relation should be discussed in the methodology of each subject area. When working on a specific problem it is, therefore, necessary to be aware of the limited interpretation of the results imposed by the conceptual isolation in the procedure.

In this thesis the operations consist first of deriving the supply and demand functions and second, the testing for homogeneity. The model and variables defy simple interpretation, so a separate section is devoted to their explanation.

Fortunately, it is possible to express the hypothesis - that the supply and demand depend on the real wage - in numerical form in terms of the properties of homogeneous functions. Tintner (44, p. 89) has shown that there are methods available for testing for homogeneity in the derived functions. The derived functions will also provide another measure of the wage elasticity of supply and demand. These elasticities and the sign of the slope of the labor supply function are questions of long standing.

Linear, Single Equation, Static Model

Considerable attention has been given recently to building more realistic models. The most familiar attempts are simultaneous equations and dynamic models. Very little success has been achieved in developing models in which the parameters do not enter in linear fashion.

If we can assume that the parameters in the structural relation enter in a linear fashion, we have hopes of using relatively simple regression analysis to derive empirical functions. The observations (X_i) need not enter in linear form, however. In some cases we may wish to substitute log X_i or sin X_i etc., for X_i.

The economic relationships probably become somewhat more complex when there is a large amount of aggregation. This aggregation is often necessary because of the nature of the problem, data available, convenience, and expense. Theil (42) is one of the few who has studied the effect of aggregation. Even in cases of considerable aggregation, it seems that a function that is linear in the parameters should give a reasonably good fit.

The presence of unemployment throughout the entire period indicates that we do not have to assume equilibrium on the labor market. The supply and demand equations can therefore be estimated separately. If we assume equilibrium on the labor market, we are faced with the problem of identification and would more appropriately use the simultaneous equations approach.

Some economists might contend that the unemployment in some years was very low and that it could be attributed entirely to frictional unemployment. However, it seems that labor supply could nearly always be increased by measures to increase overtime work, reduce frictional unemployment, and make use of part-time workers. If the results from the single equation approach are not plausible, the identification problem can be discussed again later on.

Dynamic models appear frequently in theoretical articles. There have also been attempts to fit dynamic models, notably the econometric models of the United States by Klein, Goldberger, and Tinbergen. However, the great bulk of analysis and empirical work is comparative statics (4).

In comparative statics we postulate an equilibrium system consisting of one or more simultaneous equations with some sort of a solution. The equations contain parameters which are regarded as determinants of the system and variables which are regarded as determined by the parameters. In the single equation approach, as used in this study, the system consists of a set of parameters (to be determined), a set of determining variables (exogenous and predetermined variables assumed to be known), and the unknown or predicted variable.

A dynamic model consists of a set of difference equations or differential equations having a set of parameters assumed to be constant, a set of variables with one or more "sequence variables", and a "solution". The solution in this case is a function of time, and it gives the value of each variable at each point of time. A dynamic model, in our case, would assume that the supply (demand) of labor depends upon past wages and prices as well as upon contemporary wages and prices.

Time is usually included as a variable in a static model. This does reduce the rigidity of the model to a certain extent. We hope that the time variable that is included does, to a large extent, represent those factors which are not otherwise included. If the changes are relatively smooth through time, the method is probably quite adequate.

A good dynamic model is undoubtedly more realistic, but before a dynamic model can be derived, we must have appropriate tools and must have good fundamental theory. Studies such as this which use a static model are attempts to improve fundamental theory.

To individuals who look to prediction as the supreme test of success, such a study as this may be disappointing. With present knowledge it does not seem likely that a workable dynamic model can be readily developed in this area.

Dynamic models are easy to make, but are too rigid once they are made.

The equations to be used have been derived from the assumptions of free competition. However, the elasticities of demand and supply will enter into the equations in a similar way under the various forms of imperfect competition. Tintner (45) has shown that we can expect homogeneity in the demand and supply of factors of production with respect to the prices under the following types of market organizations: free competition, monopoly, monopsony, monopolistic and monopsonistic competition, selling cost, price discrimination, product differentiation, and spatial competition.

Nature of Economic Variables

Definition and discussion of the variables to be included in the model are an important part of methodology. Economic variables are of such peculiar nature that a general discussion is also advisable.

Economic magnitudes can be conveniently grouped into two classes: those relating to intensity of satisfaction, emotion, or other subject state and those relating to the more objective phenomena such as prices, productivity, and consumption. There is undoubtedly some relationship between the supply of labor and the values of a laborer. We can measure this relationship only if it can be expressed as a function of measurable quantities, such as hours and dollars.

We speak of the supply and demand of labor as being a function of the wage, price index, and time; but we must realize that we are actually expressing some subjective quantities as a function of the objective quantities of wage, price, and time. This complex problem of identifying and measuring subjective quantities makes our problem difficult.

When the results of a statistically-fitted function are evaluated, the relation between the subjective and objective quantities must be considered. It is here that the intuitive or deductive reasoning plays an important role. With the proper blend of the deductive and intuitive, the most satisfactory results will be achieved.

In addition to the complexity of economic variables, there is a lack of data on the objective magnitudes. To obtain continuous series, it is often necessary to splice different series, to interpolate between census years, and to use data that involves a high level of aggregation. Much of the data is collected by the government and other agencies that are not concerned with theoretical analysis. The data are, therefore, not collected with the questions of economic theory in mind. Consequently, the use of data in economic research is really a by-product. However, it is of no avail to bemoan the lack of appropriate data. We must work with what is available if we plan to make progress in this important area.

There is current interest in developing statistical techniques for dealing with errors in the variables. In most studies the assumption is made that there are only errors in the equation. These errors are assumed to arise because variables which exert an influence on the dependent variable are not included in the equation or because the dependent variable is a random variable (subject to error). If all the variables are subject to error, we cannot make the assumption that there are only errors in the equation.

The method of weighted regression described by Tintner (43) is designed to deal with the situation where all variables are subject to error. Unfortunately, there is no method available for treating both errors in the equation and errors in the variables at the same time. The methods of both ordinary regression and weighted regression are used in fitting the supply and demand equation in this investigation. The results will provide another opportunity for comparing the two approaches.

There are two possible sources of data - cross-section and time series. The choice of the most appropriate source is governed by the nature of the subject. The objective is to use data on a particular population that will be most appropriate for making inferences about the topic of interest.

In the case of labor supply the time series is preferred over cross-section data. It is not meaningful to speak of labor supply to a certain geographical area or particular industry within a country. The movement between areas and occupations and the heterogeneity of different sections is too great. Unemployment cannot be easily attributed to any particular segment.

A country is the smallest meaningful unit for our interest as a labor supply. Immigration between countries is relatively small compared to movement within countries. However, the structure of the labor market is believed to vary considerably among various countries (39). For this reason it is advisable to use time series data for a particular nation rather than a cross-section study of different nations.

We realize that the market structure probably changes to a certain extent from year to year. A static model imposes certain artificial restrictions in this case. It is best to make estimates for periods of relatively short length so that a changing market structure will not impose too much of a problem.

On the demand side the appropriate unit for study is a particular industry. It is better to compare the demand for labor by a particular industry for various years rather than compare the demand of different industries. Little data is available on individual industries for any considerable time span. The most complete data are on manufacturing industries, coal mining, and, of course, the total demand for all industries (i.e., total employed). The bituminous coal industry is most appropriate for this study because labor is such a high percentage of the total costs (38, p. 87). The effect of wage changes would be the greatest in these industries in which labor is a significant part of the cost of operations. A preliminary study of the total labor demand and the demand by manufacturing industries was made, but the results from the empirical fits were not meaningful.

The next step is the selection of variables to include in the model. It is not possible to include all variables that might have an influence on the particular phenomenon of interest. We must also decide what measure of each variable is best and what level of aggregation is most desirable. In some cases the available data does not allow any choice.

The supply function as set up includes a measure of the supply of labor, Consumers' Price Index (CPI), wage rate, time, and a dummy variable for war years. The demand function includes a measure of the demand for labor, wholesale price index, wage rate, time, and a dummy variable for war years. Under this condition we assume that deflating the wage by the CPI will give the real wage to the workers, and deflating the wage by the wholesale price index will give a measure of the real wage to the employer.

To take into consideration a changing population, the supply was expressed in percentage terms,

$$S = \frac{(\text{no. employed} + \text{no. unemployed}) \times (\text{av. hrs. worked per week})}{(\text{pop. 15-64}) \times (\text{40 hrs. per week})}$$

Since we are using national figures for supply, we have no choice but to use the average wage for all employed and the CPI as it is calculated by the bureau of labor statistics.

The demand for labor by the bituminous coal industries was expressed in total hours:

$$D = (\text{no. employed}) \times (\text{av. hrs. worked per week})$$

The demand equation was also fitted with the demand expressed as:

$$D = \frac{(\text{no. employed}) \times (\text{av. hrs. worked per week})}{(\text{pop. 15-64}) \times (\text{40 hrs. per week})}$$

The multiple correlation coefficient for the regression equation using the latter as a measure of demand was very low. A much better fit was obtained when demand was measured in total hours.

All variables and a description of the calculations performed appear in Appendices B and C.

STATISTICAL METHODS USED

Regression analysis is used extensively in econometrics. In many economic applications it seems to be the most promising of statistical techniques. Ordinary multiple regression can be used to estimate the relationship between a dependent variable Y and a set of independent variables to give the best estimate of Y (i.e., unbiased and smallest variance). If we are interested in estimating the structural relationship existing in the population rather than merely predicting the dependent variable, the method of ordinary regression is quite likely to break down. If we make the plausible assumption of errors of observation in all variables, we cannot use ordinary regression. Weighted regression as outlined by Tintner (43) is designed to deal with situations where it is assumed that all variables are subject to error of observation. This method was applied in this investigation to obtain an estimate of the structural relationship in the supply and demand functions for labor.

Both ordinary multiple regression and weighted regression were used to obtain estimates of the supply and demand equations. To test the hypothesis that the supply and demand depend on the real wage, a linear restriction was imposed on the regression coefficients for the log wage and log price index. This is equivalent to forcing the equation into the form where the supply and demand are a function of the real wage. Tests were made to determine if there was a significant difference between the two fits.

Classical Least Squares

The functions to be fitted are:

$$S = b_0 + b_1W + b_2I + b_3T + b_4Pw$$

$$D = a_0 + a_1W + a_2I' + a_3T + a_4Pw$$

where S is log of labor supply, W is the log wage rate, I is the log Consumer Price Index, T is time, Pw is a dummy variable designating war and non-war years, D is the log of demand for labor, and I' is the wholesale price index.

In the method of classical least squares we minimize the sum of squares of deviations:

$$\sum_{i=1}^{N}(S_i - b_0 - b_1W_i - b_2I_i - b_3T_i - b_4Pw_i) = \sum e_i^2$$

Differentiating the left hand side with respect to the b_i the set of five normal equations is obtained. The regression coefficients and constant are computed from the normal equation. To predict supply and demand, we need only to assume that the e_i are noncorrelated and have the same variance. If tests of significance and confidence limits for the b_i are desired, we must assume that the e_i are normally and independently distributed. Ordinary regression is appropriate under the above assumptions.

Ordinary regression will yield the best estimates of the constant and regression coefficients when the e_i arise because of one of two special cases: (1) only the dependent variable is a random variable (subject to error), and the independent variables are fixed values (not subject to error); (2) there are other variables which exert an influence on the dependent variable but which are not included in the regression equation and produce the error term.

The procedure above holds only in the case of a single equation. Because of existing unemployment in all periods we assume that equilibrium does not exist on the labor market, and, therefore, the problem of identification does not arise.

The variables, S, D, W, and I, are expressed in log form so that the elasticities of supply and demand with respect to wages and prices can be obtained directly from the regression coefficients. The third regression coefficient gives the effect of exponential time trend, and the 4th coefficient is a measure of the effect of war.

The observations need not enter as linear functions in order to apply the Markoff theorem (least squares method). Log X_i, X_i^n, or sin X_i, etc., may be substituted for X_i. The estimates must be linear; in this case the regression coefficients (parameters to be estimated) must enter in a linear fashion. The linear estimates are unbiased in the sense that the mathematical expectations of the estimates are equal to the population values. The regression coefficients are the best linear estimates in the sense that the least squares estimates have the smallest variance among all linear unbiased estimates.

Test for Linear Relation

To test the hypothesis that supply and demand are homogeneous with respect to the wage and price index, we impose a linear restriction on the regression coefficients:

$$b_1' + b_2' = 0$$

We denote the new regression coefficients which are to be fitted under the linear restriction as b_1', b_2', b_3', and b_4'. Under this restriction the supply and demand are regressed on the ratio of the wage to the price index, i.e., the supply and demand are expressed as a function of the real wage.

The quantity to be minimized now becomes:

$$\sum_{i=1}^{N} S_i - b_0' - b_1'W_i - b_2'I_i - b_3'T_i - b_4'Pw_i + \lambda(b_1' + b_2')$$

where λ is a constant, the so-called Lagrange multiplier. The normal equations to determine the weighted regression coefficients and λ are now:

$$\begin{aligned}
b_1'S_{22} + b_2'S_{23} + b_3'S_{24} + b_4'S_{25} + \lambda &= S_{12} \\
b_1'S_{32} + b_2'S_{33} + b_3'S_{34} + b_4'S_{35} + \lambda &= S_{13} \\
b_1'S_{42} + b_2'S_{43} + b_3'S_{44} + b_4'S_{45} + 0 &= S_{14} \\
b_1'S_{52} + b_2'S_{53} + b_3'S_{54} + b_4'S_{55} + 0 &= S_{15} \\
b_1' + b_2' + 0 + 0 + 0 &= 0
\end{aligned}$$

where S_{ij} is the sums of squares of deviations from the means.

This set of equations is very similar to the normal equations in ordinary multiple regression and can be solved by similar methods. The sum of squares of the residuals becomes:

$$Q_2 = S_{11} - b_0' - b_1'S_{12} - b_2'S_{13} - b_3'S_{14} - b_4'S_{15} - (0)\lambda$$

The last term, of course, drops out because of the zero coefficient. The residual sum of squares for the regression equation fitted without the restriction is:

$$Q_1 = S_{11} - b_0 - b_1S_{12} - b_2S_{13} - b_3S_{14} - b_4S_{15}$$

Q_1 is distributed like χ^2 with N-p degrees of freedom. Q_2 is distributed like χ^2 with N-p+1 degrees of freedom.

We want to test the hypothesis that in the population $Q_1 = Q_2$. In other words, the hypothesis is that in the population there is actually a linear relationship between the regression coefficients such that the sum of the coefficients for log wage and log price is equal to zero.

The test function becomes:

$$F = \frac{(Q_2 - Q_1)\ (N - p)}{Q_1}$$

This is distributed as Snedecor's F with 1 and N-p degrees of freedom (44, p. 91). p is the total number of variables in our equation. The level of significance is, of course, arbitrary, but for the purposes of this paper we will reject the hypothesis that labor supply and demand are a function of the real wage if the test value is significant, at the 5 or 1 percent level. If the test value is not significant, we conclude that it is quite likely that labor supply and demand are a function of the real wage.

Weighted Regression

Estimates of the structural relationships are needed to answer the problem posed in this work. The assumptions necessary to obtain the best estimate of supply and demand from the given set of independent variables by ordinary regression might hold, but it is unlikely that the assumptions necessary for obtaining the best estimates of the structural parameters are fulfilled.

It is reasonable to assume that the measurement of economic variables is subject to a great amount of error. The variables that we observe (X_{it}) are made up of the true or systematic part (M_{it}) plus an error (Z_{it}). Oskar Morgenstern (29) presents substantial evidence for the belief that the errors of observation in such variables as wage rate, price index, supply, and demand might well be as high as 30 percent.

Under certain assumptions about the errors, Z_{it}, we can obtain estimates of the structural relationship existing between this set of variables. We neglect the errors in the equations (e_i). We also assume that the errors of observation (Z_{it}) are independent and normally distributed.

For the method of weighted regression we must know the error variance-covariance matrix. The error variances were estimated by the variate difference method (48). The computations and estimates appear in Appendix A. The error covariances were assumed to be zero.

The probabilities for the standard error ratio R_k are computed under the assumption of a normal distribution with mean zero and variance one. The assumption is valid in the case of large samples, so we might be skeptical of the results in this case because of the shortness of the period. Error variances were obtained for two periods, 1900-1928 and 1929-1955, because of the large difference in method of collecting data prior to 1929.

The choice of significant level for R_k is arbitrary. The procedure of selecting K_0 is as follows: We find a K_0 such that R_{k_0+1} is significant, but R_{k_0} is not. The systematic part of our empirical series is then assumed to be eliminated in the k_0th difference series. The variance of the k_0th finite difference series V_{k_0} is used as an estimate of the error variance. If the errors, Z_{it}, follow a linear scheme of autoregression, the variate difference method is not applicable for determining the error variances. Much work remains to be done in this area.

The variance of the 6th finite difference was used as a measure of the error variance for the supply for 1900-1928. The variance of the 4th finite difference was used as a measure of the error variance for the wage rate. The other finite differences that were used are indicated in the table.

Following the procedure given by Tintner (43), we adjust the variance-covariance matrix of the variables as follows:

$$\begin{vmatrix} a_{11} - \lambda V_1 & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} - \lambda V_2 & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} - \lambda V_3 & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{vmatrix} = 0$$

The a_{ij} are the variances and covariances of the original data, and the V_i are the error variances. The 4th and 5th variables (time, war years) are assumed to be measured without error, so V_5 and V_6 are zero.

The determinantal equation above ($A_{ij} - \lambda V_i = 0$) is solved (44, Appendix A. 2.4) to determine the values of λ. The λ are called the characteristic or latent roots; we will denote the smallest root by λ_1, the next smallest by λ_2, and the largest by λ_3.

The above determinantal equation is also used for a test for multicolinearity (i.e., to determine the number of independent linear relationships between M_{1t}, M_{2t}, . . . M_{pt} in the population which corresponds to our sample). The results appear in Table 3. The test function is:

$$\Lambda_r = (N-1) \sum_{i=1}^{r} \lambda_i$$

Under certain conditions Λ_r is distributed like χ^2 with $(N-1-p+r)r$ degrees of freedom. If Λ_1 is not significant but Λ_2 is significant, we conclude that there is one linear independent relation between the systematic parts of these variables in the population from which the sample was taken. If Λ_2 is not significant, there are two or more linear independent relations, and we are then confronted with the problem of multicollinearity.

The following system of equations is solved to obtain the weighted regression coefficients:

$$\begin{aligned}
b_1'(a_{22} - \lambda_1 V_2) &+ b_2' a_{23} &+ b_3' a_{24} + b_4' a_{25} &= a_{21} \\
b_1' a_{32} &+ b_2'(a_{33} - \lambda_1 V_3) &+ b_3' a_{34} + b_4' a_{35} &= a_{31} \\
b_1' a_{42} &+ b_2' a_{43} &+ b_4' a_{44} + b_5' a_{45} &= a_{41} \\
b_1' a_{52} &+ b_2' a_{53} &+ b_4' a_{54} + b_5' a_{55} &= a_{51}
\end{aligned}$$

The square of the standard error ratio of k_i is:

$$s_i^2 = \frac{\lambda_1 c_{ii} \sum_{n=1}^{3} \sum_{r=1}^{3} k_n k_r V_{nr}}{N - p}$$

where $k_1 = -1$ and p is the number of variables, c_{ii} is the diagonal element of the inverse matrix (A^{-1}) used to compute the b_i. (The inverse of a matrix A is denoted by A^{-1}, and $A^{-1} \cdot A = I$ where I is the unit matrix.) The ratio k_i/s_i is approximately distributed like Student's t with N-p degrees of freedom. This ratio is used to establish confidence limits and to test the null hypothesis that the coefficient b_i is not significantly different from zero.

In this study we are particularly interested in the null hypothesis that the coefficient for wage is zero (i.e., the supply of labor does not depend on the wage rate).

When the restriction that the sum of the coefficients for the wage rate and price index must be zero is imposed, the adjustment of the variance-covariance matrix becomes:

$$\begin{vmatrix} a_{11} - \lambda^{*}V_1 & a_{12} & a_{13} & a_{14} & a_{15} & 0 \\ a_{21} & a_{22} - \lambda^{*}V_2 & a_{23} & a_{24} & a_{25} & 1 \\ a_{31} & a_{32} & a_{33} - \lambda^{*}V_3 & a_{34} & a_{35} & 1 \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{vmatrix} = 0$$

The λ_i^{*} and Λ_i^{*} are computed in the same manner as λ_i and Λ_i above.

We now determine if the difference between the sums of squares fitted with the restriction Λ_1^{*} and the sums of squares fitted without the restriction Λ_1 is significant. The test function is that described by Tintner (44, p. 131):

$$F = \frac{(\Lambda_1^{*} - \Lambda_1)\ (N-p)}{\Lambda_1}$$

where F is distributed as Snedecor's F with 1 and N-p degrees of freedom. If the F is significant at the chosen level, the null hypothesis that the difference $(\Lambda_1^{*} - \Lambda_1)$ has arisen by chance must be rejected. If the F is not significant, we conclude that it is quite probable the supply and demand are a function of the real wage.

The weighted regression coefficients under the restriction that $b_1 = -b_2$ are computed from the following set of equations:

$$
\begin{aligned}
& b_1^*(a_{22} - \lambda_1^* V_2) + b_2^* a_{23} + b_3^* a_{24} + b_4^* a_{25} + u = a_{21} \\
& b_1^* a_{32} + b_2^*(a_{33} - \lambda_1^* V_3) + b_3^* a_{34} + b_4^* a_{35} + u = a_{31} \\
& b_1^* a_{42} + b_2^* a_{43} + b_3^* a_{44} + b_4^* a_{45} + 0 = a_{41} \\
& b_1^* a_{52} + b_2^* a_{53} + b_3^* a_{54} + b_4^* a_{55} + 0 = a_{51} \\
& b^* + b^* + 0 + 0 + 0 = 0
\end{aligned}
$$

where u is the Lagrange multiplier.

The b_1^* is the real wage elasticity, while the b_1 and b_2 in the previous set of equations are the elasticity of the monetary wage and price index, respectively. The results of all regression analysis appear in Table 1.

EMPIRICAL RESULTS

The supply and demand equations were fitted by the methods of weighted and ordinary regression for each of the four separate periods: 1900-1918, 1919-1928, 1929-1940, 1941-1955. The conditions varied enough between 1900 and 1955 that it was advisable to derive equations for separate sub-periods. Since the data for the years prior to 1929 were much less complete, the results for this period may not be as valid.

The frequent splicing of different series, interpolation between census years, and possibly large errors in measurement make one skeptical of the information prior to 1929. It is also conceivable, of course, that conditions in the industries changed to such an extent during this time that our model is not adequate. It is difficult to analyze conditions for a period so far back and under a situation so different from the present.

Since 1929, methods of collecting data have been more reliable and the data much more complete. Data have been collected on a larger number of factors, and they are collected annually or oftener so that interpolation is usually not necessary. Not only was there a large amount of interpolation prior to 1929, but the estimates were often made several years later. Much of the data for 1900-1929 was calculated in 1947. Sampling techniques are now being used extensively making it possible to collect much more data at an equal or less cost. We expect that more emphasis can be placed on the results from the analysis for the periods after 1929 because of the better data.

The supply equations were fitted both with and without time included as an explanatory variable. This factor seemed to have a considerable effect on the magnitude of the coefficients for wage and price index. The fairly constant rise in the wage and, to a lesser extent, the price index throughout the entire interval appeared to be the cause.

Sensible results were obtained for all fits of the supply equation. The coefficients varied between the periods but not beyond what can be expected. But on the other hand, the results from the study of demand were not meaningful for the two periods prior to 1929. Demand equations were fitted for bituminous coal industries, manufacturing industries, and all industries combined. In all cases the fits for periods prior to 1929 were very poor.

The results for all fits appear in Table 1. Estimates of the wage elasticity, price elasticity, exponential effect of time, and effect of war with the accompanying tests of significance and confidence limits are given for both methods of fitting.

Table 1. Estimates, tests of significance, confidence limits

Regression coefficient	Estimate	Test of significance (t)	95 percent confidence limits	
1900-1918				
Supply with time trend, ordinary regression				
Wage elasticity	1.0646	5.026[a]	1.5191	.6101
Price elasticity	-1.0884	3.710[a]	-1.7175	-.4593
Time trend	-.0041	3.096[a]	-.0070	-.0012
War effect	.0289	1.684	.0657	-.0079
Homogeneous supply with time trend, ordinary regression				
Real wage elasticity	1.0663	4.195[a]	1.6113	.5213
Time trend	-.0043	8.120[a]	-.0054	-.0032
War effect	.0263	3.487[a]	.0425	.0101
Supply with time trend, weighted regression				
Wage elasticity	5.5954	1.780	12.3376	-1.1468
Price elasticity	-6.0929	1.748	-13.5670	1.3812
Time trend	-.0062	.849	-.0219	.0095
War effect	.1239	1.127	.3596	-.1118
Homogeneous supply with time trend, weighted regression				
Real wage elasticity	6.6828	1.625	15.5014	-2.1357
Time trend	-.0119	1.955	-.0250	.0012
War effect	.0798	1.362	.2055	-.0459
1919-1928				
Supply with time trend, ordinary regression				
Wage elasticity	-.6852	1.897	-1.5691	.1987
Price elasticity	.3816	1.074	1.2510	-.4878
Time trend	.0036	.923	.0132	-.0060

[a]Significant at the 1 percent level

Table 1. (Continued)

Regression coefficient	Estimate	Test of significance (t)	95 percent confidence limits	
Homogeneous supply with time trend, ordinary regression				
Real wage elasticity	-.5118	1.365	-1.4293	.4055
Time trend	.0026	.509	.0149	-.0097
Supply with time trend, weighted regression				
Wage elasticity	-2.4125	.943	-8.4000	4.1150
Price elasticity	2.0406	.821	8.1263	-4.0451
Time trend	.0218	.803	.0882	-.0446
Homogeneous supply with time trend, weighted regression				
Real wage elasticity	2.0022	1.065	6.6006	-2.5962
Time trend	-.0242	1.223	-.0711	.0228
Supply without time trend, ordinary regression				
Wage elasticity	.3387	3.263[b]	-.5927	-.0947
Price elasticity	.0887	.595	.4537	-.2773
Homogeneous supply without time trend, ordinary regression				
Real wage elasticity	-.2908	2.510[b]	-.5743	-.0063
1929-1940				
Supply with time trend, ordinary regression				
Wage elasticity	-1.2286	2.712[b]	-2.2736	-.1836
Price elasticity	1.7126	3.460[a]	2.8541	.5711
Time trend	.0141	2.205	.0288	-.0006
Homogeneous supply with time trend, ordinary regression				
Real wage elasticity	-.7218	.918	-2.5358	1.0922
Time trend	.0044	.407	.0291	-.0203

[a]Significant at the 1 percent level

[b]Significant at the 5 percent level

Table 1. (Continued)

Regression coefficient	Estimate	Test of significance (t)	95 percent confidence limits	
Supply with time trend, weighted regression				
Wage elasticity	-1.7727	6.674[a]	-2.3852	-1.1602
Price elasticity	2.3052	5.704[a]	3.2371	1.3733
Time trend	.0217	5.200[a]	.0229	.0205
Homogeneous supply with time trend, weighted regression				
Real wage elasticity	-1.2389	2.491[b]	-2.3320	-.1436
Time trend	.0113	2.410[b]	.0221	..0005
Supply without time trend, ordinary regression				
Wage elasticity	-.2459	2.300[b]	-.4877	-.0041
Price elasticity	.6407	5.762[a]	.8922	.3892
Homogeneous supply without time trend, ordinary regression				
Real wage elasticity	-.4053	4.018[a]	-.6339	-.1768
1941-1955				
Supply with time trend, ordinary regression				
Wage elasticity	.6818	1.92	1.4736	-.1100
Price elasticity	-.2390	1.53	-.6391	.1611
Time trend	-.0149	2.12	-.0306	.0008
War effect	.0552	10.22[a]	.0672	.0432
Homogeneous supply with time trend, ordinary regression				
Real wage elasticity	-.0771	.562	-.3022	.2646
Time trend	.0007	.529	.0038	-.0023
War effect	.0527	8.125[a]	.0661	.0392

[a]Significant at the 1 percent level

[b]Significant at the 5 percent level

Table 1. (Continued)

Regression coefficient	Estimate	Test of significance (t)	95 percent confidence limits	
Supply with time trend, weighted regression				
Wage elasticity	.7378	.466	4.2625	-2.7869
Price elasticity	-2.0951	2.532[b]	-3.9418	-.2444
Time trend	.0367	3.665[a]	.0591	.0143
War effect	-.1691	12.076[a]	-.2002	-.1378
Homogeneous supply with time trend, weighted regression				
Real wage elasticity	-.2822	2.236[b]	-.5652	.0010
Time trend	-.0132	12.573[a]	-.0156	-.0109
War effect	-.1982	17.240[a]	-.2238	-.1726
Supply without time trend, ordinary regression				
Wage elasticity	-.0574	.749	-.2261	.1113
Price elasticity	.0886	.848	.3186	-.1414
War effect	.0533	8.717[a]	.0668	.0398
Homogeneous supply without time trend, ordinary regression				
Real wage elasticity	-.0118	.204	-.1391	.1155
War effect	.0518	8.554[a]	.0651	.0385
1900-1955				
Supply with time trend, ordinary regression				
Wage elasticity	-.1874	1.085	-.5340	.1592
Price elasticity	.2871	1.987	.5770	-.0028
Time trend	-.0024	1.328	-.0060	.0012
War effect	.0044	3.369[a]	.0709	.0179

[a]Significant at the 1 percent level

[b]Significant at the 5 percent level

Table 1. (Continued)

Regression coefficient	Estimate	Test of significance (t)	95 percent confidence limits	
Homogeneous supply with time trend, ordinary regression				
Real wage elasticity	-.3782	2.717[a]	-.6575	-.0989
Time trend	.0001	.088	.0024	-.0022
War effect	.0395	2.976[a]	.0662	.0128
Supply without time trend, ordinary regression				
Wage elasticity	-.4089	2.841[a]	-.6977	.1199
Price elasticity	.4502	1.841	.9414	-.0410
War effect	.0380	.965	.1169	-.0409
Homogeneous supply without time trend, ordinary regression				
Wage elasticity	-.3662	4.283[a]	-.5381	-.1943
War effect	.0399	1.011	.1169	-.0409
1929-1940				
Demand with time trend, ordinary regression				
Wage elasticity	.0234	.03	1.7428	-1.6960
Price elasticity	.8536	.94	2.9511	-1.2439
Time trend	-.0211	3.01[b]	-.0372	-.0050
Homogeneous demand with time trend, ordinary regression				
Real wage elasticity	.7430	.242	7.8178	-6.3318
Time trend	-.0190	1.969	-.0414	.0032
Demand with time trend, weighted regression				
Wage elasticity	-1.2555	.776	-4.9864	2.4758
Price elasticity	2.4157	1.236	6.9232	-2.0918
Time trend	-.0132	1.177	-.0391	.0127

[a]Significant at the 1 percent level

[b]Significant at the 5 percent level

Table 1. (Continued)

Regression coefficient	Estimate	Test of significance (t)	95 percent confidence limits	
Homogeneous demand with time trend, weighted regression				
Real wage elasticity	-4.5425	1.194	-13.3134	4.2284
Time trend	-.0500	1.551	-.1243	.0243
1941-1955				
Demand with time trend, ordinary regression				
Wage elasticity	-1.7488	2.04	-3.6563	.1587
Price elasticity	1.7269	5.40[a]	2.4405	1.0133
Time trend	-.0154	.80	-.0584	.0276
War effect	-.0514	.95	-.1720	.0692
Homogeneous demand with time trend, ordinary regression				
Real wage elasticity	-1.7221	6.038[a]	-2.3576	-1.0868
Time trend	-.0160	3.308[a]	-.0268	-.0052
War effect	-.0501	4.227[a]	-.0765	-.0237
Demand with time trend, weighted regression				
Wage elasticity	-5.8804	1.996	-12.0089	2.4811
Price elasticity	2.9036	3.151[a]	4.8203	.9869
Time trend	.0742	1.144	.2092	-.0608
War effect	-.1873	1.405	-.4844	.1098
Homogeneous demand with time trend, weighted regression				
Real wage elasticity	-2.0176	6.260[a]	-2.6879	-2.6529
Time trend	-.0125	2.336[b]	-.0233	-.0015
War effect	-.0288	.717	-.1122	.0548

[a]Significant at the 1 percent level

[b]Significant at the 5 percent level

Table 1. (Continued)

Regression coefficient	Estimate	Test of significance (t)	95 percent confidence limits	
		1929-1955		
		Demand with time trend, ordinary regression		
Wage elasticity	-.9336	2.255[b]	-1.9747	-.0725
Price elasticity	1.8854	23.408[a]	1.9352	1.8356
Time trend	-.0206	3.080[a]	-.0345	-.0067
War effect	.1177	39.107[a]	.1240	.1114
		Homogeneous demand with time trend, ordinary regression		
Real wage elasticity	-1.8301	53.828[a]	-1.9008	-1.7594
Time trend	.0005	.026	.0411	-.0401
War effect	.0796	31.076[a]	.0849	.0742
		Demand with time trend, weighted regression		
Wage elasticity	-2.6450	3.097[a]	-4.4213	-.8687
Price elasticity	2.8694	4.712[a]	4.1361	1.6027
Time trend	-.0029	.251	-.0258	.0210
War effect	.0821	1.926	.1716	-.0066
		Homogeneous demand with time trend, weighted regression		
Real wage elasticity	-3.3397	4.753[a]	-4.8011	-1.8783
Time trend	.0046	1.527	.0107	-.0022
War effect	.0672	1.436	.0165	-.0302

[a]Significant at the 1 percent level

[b]Significant at the 5 percent level

Estimates by Ordinary Regression

The multiple correlation coefficients for the supply relationships were significant at the 1 percent level for all periods except 1919-1928. The same was true for the fitted equations where time was omitted as an explanatory variable. The tests of significance show that the coefficient for time was only significant for the period 1900-1918.

The results show a negative elasticity of supply for the second, third, and overall periods; and a positive elasticity for the first and fourth periods. However, the coefficients for the wage and cost-of-living index in the fourth period were not significant at the 5 percent level. The supply equation without time trend shows a negative elasticity of supply.

Most economists agree that we should expect a negative elasticity of supply in the more recent periods. It is, however, not unlikely that the shape of the supply curve could change depending upon the wage level, composition of the labor force, and the presence of war. The results obtained by Douglas from both cross-section and time series analysis indicated a negative supply curve for labor.

It is conceivable that there was a zero wage elasticity for some of the periods. The regression coefficient for wage rate was significantly different from zero for the periods 1900-1918 and 1929-1940 only. In the other periods the wage elasticity was probably zero or close to zero.

The coefficient measuring the effect of war was highly significant in the fourth and overall periods. The same coefficient for the period 1900-1918 was not significant.

On the demand side significant fits were obtained for the periods 1929-1940, 1941-1955, and 1929-1955. In the first period the only significant regression coefficient was for the time trend. The results show a positive wage elasticity, but little emphasis can be placed on the coefficient because the confidence limits range from -1.69 to 1.74. A positive wage elasticity is, of course, unrealistic because it indicates the minimizing rather than the maximizing of profits by the employers (54).

The coefficient for wage is only significant where the equation was fitted for the overall period. The coefficient for price index was significant in the second and overall period. The war effect and time trend in the first period were highly significant.

Tests for Homogeneity

The homogeneous functions are the equations fitted under the restriction that the sum of the coefficients for wage and price index must be zero. In this form the functions are homogeneous of degree zero with respect to the wage rate and price index. The common coefficient is expressed as the real wage elasticity (*i.e.*, the coefficient for the ratio of the wage to the consumer price index).

The test for homogeneity is applied to the ratio of the "residual sum of squares for the equations fitted with the restriction" to the "residual sum of squares for the equation fitted without the restriction". If the difference between these sums of squares is so large that it is not likely that it could have arisen by chance, we must reject the hypothesis that there is no difference between the fits. The results of the analysis variance tests are given in Table 2.

The tests values are not significant for the first, second, and overall period. The test value is significant at the 5 percent level for 1941-1955 and significant at the 1 percent level for 1929-1940. For the equations fitted without a time trend the values are non-significant in all periods except 1929-1940. Including time as an explanatory variable appears to change the wage and price index coefficients enough to affect the test for homogeneity.

On the demand side the F value was not significant in the second period, but it was significant at the 5 percent level in the first period and at the 1 percent level in the overall period.

In those instances where the test value is significant, the null hypothesis must be rejected. This indicates that on the basis of this analysis we cannot assume that the function

Table 2. Analysis of variance tests[a]

	Period	Without restrictions	With restrictions	Variance ratio (F)
Weighted sums of squares				
Supply with time trend	1900-1918	5.520	5.580	.097
Supply with time trend	1919-1928	3.720	6.700	4.210
Supply with time trend	1929-1940	4.360	7.150	4.450
Supply with time trend	1941-1955	3.550	4.620	3.100
Supply with time trend	1900-1955			
Residual sums of squares				
Supply with time trend	1900-1918	.910	.911	.015
Supply with time trend	1919-1928	.614	1.048	4.240
Supply without time trend		.817	1.092	2.020
Supply with time trend	1929-1940	.988	3.240	15.950[b]
Supply without time trend		1.589	3.031	7.490[c]
Supply with time trend	1941-1955	7.865	11.840	5.054[c]
Supply without time trend		11.390	12.169	.684
Supply with time trend	1900-1955	33.994	36.132	3.208
Supply without time trend		35.198	36.122	1.339

[a] $F = \frac{(\Lambda_1^* - \Lambda_1)(N - p)}{\Lambda_1}$ and $F = \frac{(Q_2 - Q_1)(N - p)}{Q_1}$ are distributed as Snedecor's F (variance ratio) with 1 and $(N - p)$ degrees of freedom.

[b] Significant at the 1 percent level

[c] Significant at the 5 percent level

Table 2. (Continued)

	Period	Without restrictions	With restrictions	Variance ratio (F)
		Weighted sums of squares		
Demand with time trend	1929-1940	11.180	14.441	2.320
Demand with time trend	1941-1955	5.180	7.130	3.760
Demand with time trend	1929-1955	29.060	32.980	2.970
		Residual sums of squares		
Demand with time trend	1929-1940	18.280	35.390	7.427[c]
Demand with time trend	1941-1955	.771	.772	.013
Demand with time trend	1929-1955	67.730	100.270	10.570[b]

[b]Significant at the 1 percent level

[c]Significant at the 5 percent level

is homogeneous with respect to wages and prices.

Results can be better interpreted after the equations from the weighted regression method are analyzed.

Estimates by Weighted Regression

The first interesting results in the procedure of weighted regression occur in the test for multicollinearity (Table 3). The number of linear relationships existing in the population has an important bearing in determining the

Table 3. Test for multicollinearity[a]

	Period	Latent root (λ_i)	Chi-square value
Supply with time trend	1900-1918	1 = .290 2 = 2.290	5.220 43.800[b]
Supply with time trend	1919-1928	1 = .413 2 = 2.330	3.720 24.680[c]
Supply with time trend	1929-1940	1 = .370 2 = 2.434	4.360 31.340[b]
Supply with time trend	1941-1955	1 = .252 2 = 1.256 3 = 19.293	3.528 26.670 212.223[c]
Supply with time trend	1900-1955	1 = 5.427	261.800[c]
Demand with time trend	1929-1940	1 = 1.016 2 = 1.925	11.176 32.351[b]
Demand with time trend	1941-1955	1 = .370 2 = 1.990 3 = 17.930	5.180 33.040 284.058[c]
Demand with time trend	1929-1955	1 = 1.816 2 = 5.594	29.056 118.560[c]

[a] $\Lambda_r = (N-1) \sum_{i=1}^{r} \lambda_i$ is distributed as chi-square with $(N-1-p+r)r$ degrees of freedom.

[b] Significant at the 5 percent level.

[c] Significant at the 1 percent level.

proper method of analysis (42, pp. 33, 127). It is also important from the theoretical standpoint. The test value (χ^2) for estimating the number of linear relationships is computed from the latent roots.

Except for 1941-1955 the tests indicate that there is not more than one linear independent relation between the set of variables included in the supply relation and the demand relation. In these cases the test value computed on the basis of the first and second latent root was significant at the 5 percent level. For 1941-1955 the first two test values computed from the supply relation are not significant, but the third test value is significant. This indicates two linear independent relations.

The test values computed from the demand relation are more difficult to interpret. The second test value is actually not significant at the 5 percent level, but it is very close to the tabulated value (33.0 : 33.9). The test value does indicate the possibility of two independent linear relations.

The result in the period 1941-1955 may have been caused by the small amount of unemployment, (indicating equilibrium), or it may have been caused by the nearly constant rise in the wage and price index throughout this period. If the result were caused by the former, we would expect it less likely on the demand side. The test values tend to confirm this. The total demand for labor might equal total supply, but it is less likely that the demand for labor by an individual industry would be equal to the potential supply to that industry.

For the overall period in the supply relation all test values are significant. This suggests that there is no linear relation that holds throughout the entire period. If Λ_1 is significant it indicates that there are zero independent linear relations. This further indicates that the supply relation should be analyzed for short periods.

The magnitude of the weighted regression coefficients for the latter periods agree fairly closely with the coefficients derived by ordinary regression. In all cases they are a little higher, however.

For the supply relation the results show a negative wage elasticity for 1919-1928, 1929-1940 and a positive elasticity for the periods 1900-1918, 1941-1955. This was the same as in ordinary regression. However, in the case of weighted regression, the coefficient for the wage was not significant in the latter two instances. This indicates that the supply may not depend on the wage rate at all (zero elasticity).

A wage elasticity of 5.6 and -2.4 for the first two periods seem higher than we would normally expect. Little significance can be attached to these coefficients since the standard errors are high and consequently, the fiducial limits very wide.

On the demand side the wage elasticity is negative for all three periods. The real wage elasticity varies between -2.0 for 1941-1955 and -4.5 for 1929-1940. It is only in the second and overall period that the tests indicate the wage coefficient significantly different from zero.

Tests for Homogeneity

In the weighted regression method the difference between the fits in both the supply and demand equations were not significant at the 5 percent level in any of the periods. The weighted sums of squares and variance ratios (F) are given in Table 2.

The F values computed from the supply equations are highest for 1919-1928 and 1929-1940, but even in these periods the values are well below the tabulated values for the 5 percent level. In the ordinary regression the test value was highly significant for the period 1929-1940.

In neither the supply nor the demand relations is it necessary to reject the hypothesis that the equations are homogeneous with respect to wages and prices. It is quite likely that both the supply and demand depend on real wage (*i.e.*, they depend on the ratio of the wage to the price index).

INTERPRETATIONS AND CONCLUSIONS

Drawing inferences in an empirical science will always involve a certain risk. The pure deductive or intuitive method is not adequate because it does not rely on empirical verification or testing. No confidence level or relative frequency can be attached to statements. On the other hand, the inductive method is also risky because we cannot be sure that the sample data are representative of the population for which we would like to make inferences. There is also the problem of a complex maze of relationships in any set of sample data.

Considerable time was spent in criticizing the present predominantly deductive approach to wage theory. The interpretation of results from this empirical investigation will end with a critical evaluation of the confidence that can be placed in the results of the inductive approach. This seems to leave no alternative. However, if progress is to be made it is necessary to draw some conclusions and to work on improving methodology. The conclusions are the best inferences that can be drawn from the results of the empirical study. The procedure is useful as a study in methodology.

The choice of procedure and data was oriented toward answering one main question: Does the supply of and demand for labor depend on the real or money wage? Because of this orientation and the complex nature of this area of economics, caution must be used in drawing conclusions from these results about other aspects of wage theory. It is hoped that some information can be obtained from the derived equation about the elasticities of supply and demand.

To answer the question of homogeneity, it is necessary to study the structure of the supply and demand relation. There were no empirically derived functions available for this analysis. The most important empirical results for this country were obtained by Douglas in his voluminous labor study in the 1920's. In addition to its being dated, his work was oriented toward determining the marginal productivity of labor and the relative share of national income going to labor. The derived functions in this study are among the very few that are available (5, pp. 350-351).

If there are errors in all variables, the simpler methods of statistical analysis will not yield the structural parameters. The special technique of weighted regression was used in an attempt to obtain better estimates of the structural relations.

It is not unlikely that there are both errors in the variable and errors in the equation in the models and data used in economics. There is no method available at the present time for dealing with both types of errors. The methods of weighted and ordinary regression were both used, thereby furnishing more information for evaluating the two approaches.

Homogeneity and Elasticity of Supply and Demand

The statistical tests do not indicate that we must reject the null hypothesis that there is no significant difference between the equations fitted with and without the restriction. The difference between the fits was such that it could easily have arisen by chance. It is very probable that both the supply and demand functions for labor are homogeneous of degree zero with respect to wages and prices. Both supply and demand appear to depend on the real wage.

This conclusion is in agreement with classical theory, but is contrary to the view held by Keynes and many of his followers. Keynes held that such things as money illusion and irrational behavior were reasons for believing that the supply of labor depended on the money wage. Modigliani (27) has shown that the necessary condition for the Keynesian position, namely, that it is possible to have equilibrium without full employment, is that one of the equations is not homogeneous with respect to wages and prices.

There has been little doubt that the demand for labor depended on the real wage, but, as stated earlier, there are reasons for assuming that the supply of labor might depend on the money wage. The results of this investigation suggest that both the supply and demand for labor depend on the real wage.

The form of competition in the labor market is a topic of current interest. We are quite certain that there is considerable deviation from the classical model of perfect competition. Some form of monopoly and monopsony is more likely because of the size of industry and the uniting of labor into large collective bargaining units. Tintner (45) has shown that we can expect the supply of and demand for factors of production to be homogeneous of degree zero in these forms of competition as well as in the case of perfect competition.

Conclusions concerning supply and demand responses to changes in the real wage are difficult. The assumptions under which the model was set up are likely to be only partially fulfilled. Conditions in the future periods for which we would like to make inferences are also likely to vary considerably from the conditions that existed during the period studied. The results indicate that the supply and demand equations varied considerably for the period between 1900 and 1955.

It seems more likely that the supply of labor decreases rather than increases in response to an increase in real wages. Caution must be used in making such a statement because we have been dealing with a static model. It is conceivable that the response pattern changes with different levels of income, at different points of time, and for different groups of individuals. These points are discussed in the next chapter under suggestions for further study.

It is not inconceivable that the supply is unresponsive to changes in the real wage (zero elasticity). The empirical results suggest that the elasticity is zero or close to zero. Douglas (13) in 1929, using a cross-section study, calculated the E_s to be between -.24 and -.33. He did not give fiducial limits for his estimates. With an increase of 1 percent in the real wage, the demand for labor in an industry such as bituminous coal can be expected to decrease between 2 and 4 percent. This seems rather high, but there is evidence to support such a large figure. Douglas (13, p. 152) estimated the E_d to be between -3.0 and -4.0. The coal industry is peculiar in that labor is a high percentage of total cost (estimated to be about 60 percent) and, of course, a considerably higher percentage of the variable costs. The E_d for a factor is expected to be greater when the cost of this factor is a large portion of the total cost.

Importance of the Results

Knowledge of the homogeneity and elasticity properties of the supply and demand for labor is crucial for sound economic policy and economic theory. Effort to maintain high and stable levels of employment must begin by obtaining as reliable estimates on these two aspects as possible (9, p.

342). The best estimates can probably be obtained by studying the past. But we must realize that economic conditions vary considerably, and the future, therefore, may be quite different from the past. Consequently, the validity of inferences drawn from the study of a past period is unknown.

It would be desirable if probability statements could be related to confirmation of hypotheses rather than based on relative frequencies. Conclusions from an empirical investigation are based on analysis of data from some past period and consequently are only relevant to a period with conditions similar to the period studied. The degree to which future periods will be similar is, of course, uncertain. It is hoped that the conclusions will hold at least approximately for the present and immediate future. Extreme caution must be used in applying the conclusions in the more distant future.

The method of weighted regression was used in an attempt to obtain better estimates of the structural relationships. There seems to be reason to expect errors of observation in all variables, and, in this case the simpler method of ordinary regression breaks down. The estimates and probabilities using the method of weighted regression are valid only under the following assumptions:

(1) The theoretical framework must be valid. More specifically, there must exist a static supply and demand function in the unknown hypothetical infinite

population which are at least approximately linear in the logarithms.

(2) There must be no errors in the equations.

(3) The errors in the variables must have constant variance over time.

(4) The individual items of each error series must be independent, and the errors in one series must be independent of the errors in other series.

(5) The errors in the variables must be normally distributed.

(6) The error variance must be known.

(7) The sample must be large.

These conditions are only partially fulfilled. We do not know the extent that actual conditions deviate from the assumptions, nor do we know the subsequent effect on the conclusions.

We should not place too much confidence in our results. In methodology, a great deal is yet to be learned concerning both errors of observation and errors in the equation. Our factual knowledge is deficient about new factors in the labor market (10). The labor union undoubtedly considers a much wider range of factors in wage bargaining than does an individual (13). The role and importance of political and institutional factors in the union are uncertain. These factors are difficult to observe and, undoubtedly, vary considerably through time and from union to union. A great amount has been written on these aspects, but it is far from conclusive. More research needs to be done in this area so that we can be more certain that our models are representing the real world labor market.

Complete reliance on deductive reasoning is unlikely to lead to much further progress. On the other hand, undue confidence in any one set of empirical findings is also unwarranted. It is important that we derive new ideas from both the deductive and inductive (empirical) approach. However, these new ideas should not be emphasized to such an extent that we are unappreciative of earlier work or the results from a different approach. A blend of the empirical and intuitive approaches should produce the best results, each method serving as sort of a check on the validity of the other. When the results of the two approaches are in close agreement, we can have considerable confidence in the results; but when the conclusions differ both sets of conclusions should be re-examined. In such a complex area the testing and checking is just as important as the formulating of hypotheses.

SUGGESTIONS FOR FURTHER STUDY

The remarks in this chapter are meant to be useful in further attempts to derive supply functions for labor. They are not necessarily pertinent to the question of homogeneity. The suggestions evolved, for the most part, from the difficulties and results of empirical work for this thesis.

The discussion in the first section pertains to the possible variation in the supply curve depending upon the number of hours worked per week, the level of income, and the composition of the labor force. Since data are not available for these aspects, the discussion will proceed by breaking the supply curve into more fundamental units. It is hoped that the conclusions will be useful as guides in collecting more pertinent data and useful for analyzing available data.

The second section is a discussion of wage and employment data. The lack of good data is one of the biggest obstacles at the present time. The last section is an overall appraisal of methodology.

Analysis of Labor Supply

It has frequently been suggested that the labor supply curve changes from positive to negative slope as the wage rate increases. Certain conditions under which this will be true have been cited, but little effort has been spent to determine when and if these certain conditions are present.

Somewhat more attention has been given to the possible variance in the supply curves between different types of individual (4, p. 352). Both of these aspects are worthy of further investigation. In this chapter attention will be focused on the sign of the slope and on the numerical value of the elasticity of labor supply. We will break the labor supply function into more fundamental components and carry on the discussion in these terms (5).

Figure 1 gives a graphic presentation of the component parts assuming the wage rate is constant (W_0). uu is the marginal utility of hourly income; $\ell\ell$ the marginal utility of leisure; and dd is the curve of direct marginal disutility resulting from the work itself. $\ell d \ell d$, the sum of $\ell\ell$ and dd, shows the total marginal disutility of labor.

The hours worked per week are measured along the horizontal axis, and marginal utility is designated by the left hand ordinate. The right hand ordinate is the left hand ordinate inverted. It permits easier interpretation of $\ell\ell$ and dd. Both of these quantities can be just as effectively measured along the left hand ordinate, however.

The intersection of $\ell d \ell d$ and uu designates the number of hours that the individual will desire to work at wage W_0. Figure 1 represents a situation where an individual desires to work 40 hours and have 50 hours of leisure, while Figure 4 represents the situation where an individual does not desire

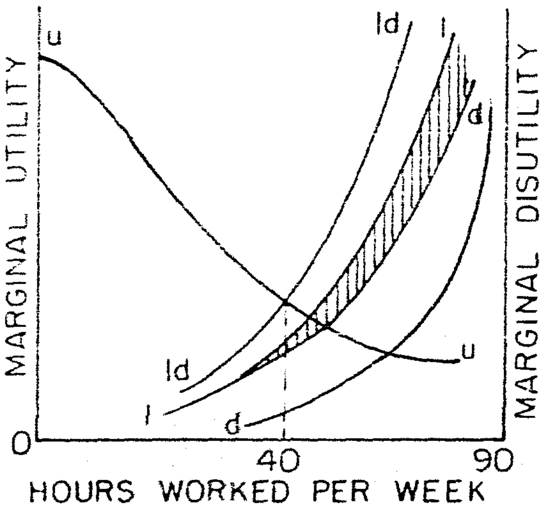

Figure 1. Males, head of household; individuals desiring to work 40 hours and have 50 hours of leisure

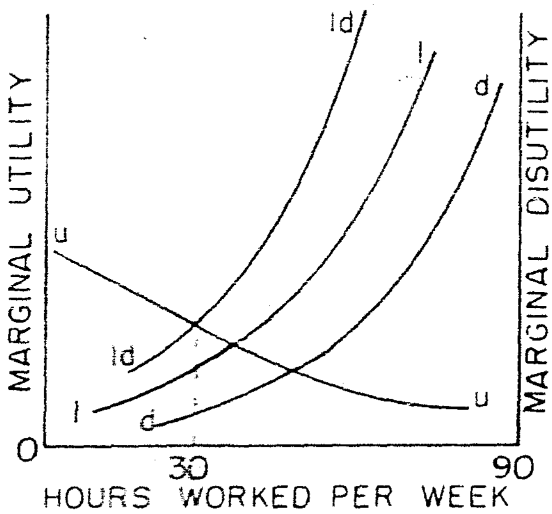

Figure 2. Individuals desiring not to work at wage rate W_o

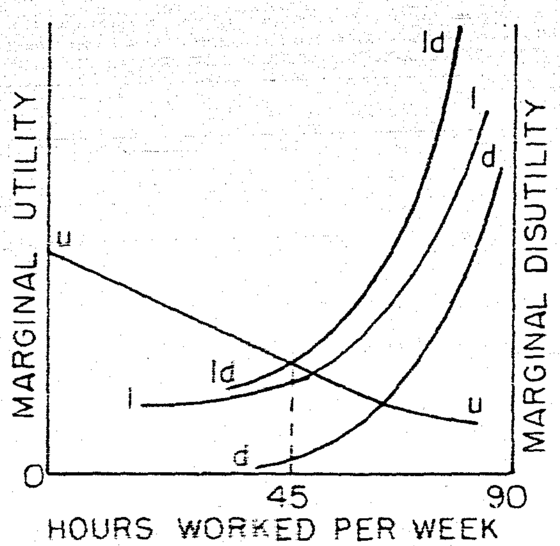

Figure 3. Members of a family unit but not head of household; individuals desiring to work 30 hours per week

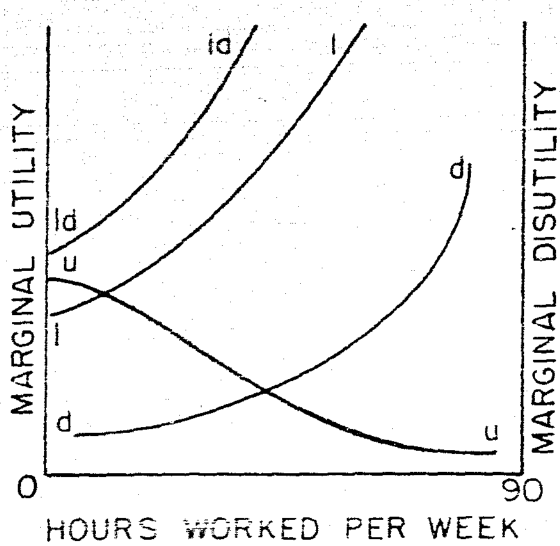

Figure 4. Single persons not members of a family unit; individuals desiring to work 45 hours per week

to work at all for a wage W_o. At the intersection of uu and $\ell d\ell d$ the classical argument states that the utility to be gained from working one additional hour is just equal to the disutility of working this additional hour. As the wage rate changes the uu and $\ell\ell$ curves will shift. The new intersection points of $\ell d\ell d$ and uu will trace out the supply curve (Figure 5).

The marginal disutility of work (dd) would not be expected to change as a result of changes in the wage rate; it is a function of the hours worked $[dd = d(H)]$. A 1 percent change in the wage rate will cause a certain percent increase or decrease in supply depending upon the shifts in $\ell\ell$ and uu.

With the wage rate constant the horizontal axis is a measure of total income, and uu is the familiar marginal utility of income. A wage change will have two effects on uu: (1) an equal percentage change in the same direction due to the change in hourly income, (2) a shift in the opposite direction because of the change in marginal utility of income.

For convenience in later use the percent change in marginal utility (uu) for a 1 percent change in income will be called E_u.

$$E_u = - \frac{du}{dI} \frac{I}{u}$$

It is reasonable to assume that the marginal utility of income decreases as income rises; hence, it is more convenient to express the elasticity of utility with respect to income

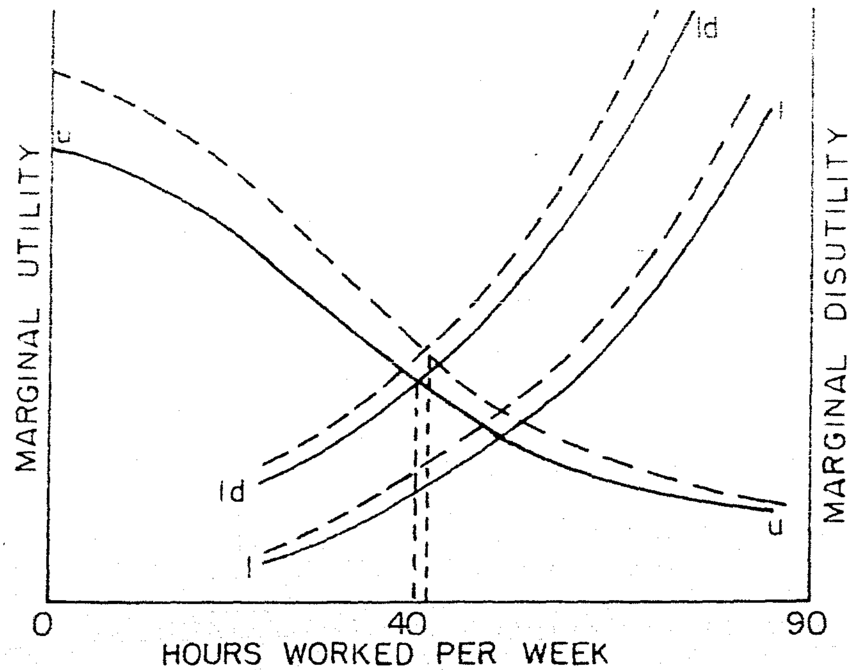

Figure 5. Wage increase from W_o to W_1

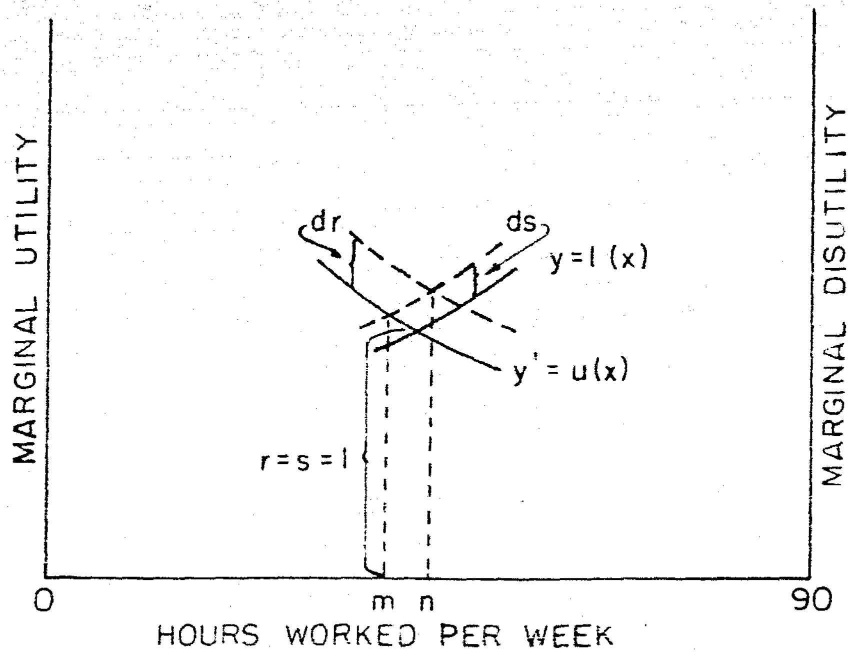

Figure 6. Change in supply for a given change in wage rate

as minus one times the derivative of utility with respect to income. The net shift in uu for a 1 percent change in wages is then equal to $1 - E_u$.

Unfortunately, we have no numerical measure of E_u. The uu curve is drawn so it is convex to the origin over most of the range. Even this is assuming some knowledge about the third derivative of utility as a function of income. It is interesting to note that in two commonly expressed functional forms for utility ($U = \log I$, $U = I^{\alpha}$ with $\alpha < 1$) the third derivative of utility with respect to income is positive making the curve concave from above as is shown.

The $\ell\ell$ curve is drawn upward sloping to the right and concave from above. This curve is composed of two functional relationships: (1) leisure as a function of income $[\ell = f(W_oH)]$, (2) leisure as a function of total hours of leisure $[\ell = g(h)]$. The shaded portion (Figure 1) represents the functional relationship of leisure on income. The lower line represents leisure as a function of the number of hours of leisure.

We need to define two properties of the $\ell d\ell d$ curve: (1) the change in ℓd as the hours of work increase and (2) the shift in $\ell d\ell d$ as the wage rate changes. ℓd can be expressed as F(H) where:

$$F = d(h) + f(W_oH) + g(H).$$

The first property will be defined in terms of the percent change in ℓd for a 1 percent change in hours and designated as $E_{\ell d}$.

$$E_{\ell d} = \frac{d\ F(H)}{d(H)} \frac{H}{\ell d}$$

The second property is defined as the percent shift in $\ell d \ell d$ for a 1 percent change in the wage rate; it will be designated as ℓd^*.

$$\ell d^* = \frac{1}{2} \frac{f(x)}{F(x)}$$

The shift in $\ell d \ell d$ is equal to the difference in utility from the change in income (1 percent) when used with n hours of leisure and when used with n-1 hours of leisure. This quantity would be expected to be quite small in most cases.

The slope of the supply curve is determined by the relative magnitudes of the shifts in the uu and $\ell d \ell d$ curves. Figure 5 shows a hypothetical situation given a 1 percent increase in the wage rate. In this case $1 - E_u > \ell d^*$, and a positive supply curve results. The supply curve will have a positive slope if $E_u + \ell d^* < 1$, a negative slope if $E_u + \ell d^* > 1$ and will be completely inelastic if $E_u + \ell d^* = 1$.

The elasticity of supply can be expressed in terms of the quantities that have been defined. If the shifts in uu and $\ell d \ell d$ are not large, the elasticity of supply (E_s) can be approximated quite simply. The symbols to be used in deriving the formula are shown in Figure 6.

$$E_s = \frac{mn}{om}$$

dr is the shift in the uu curve and ds is the shift in $\ell d \ell d$; both result from a 1 percent change in the wage rate. For simplicity the two curves are expressed as:

$$uu = y = u(x)$$

$$\ell d \ell d = y' = \ell(x)$$

At the initial equilibrium:

$$u(x) = \ell(x)$$

$$ru(x) = s\ell(x)$$

where r = s = 1. With a wage change the curves shift by the amounts dr and ds. Differentiating both sides we get in terms of first approximations:

$$u \cdot dr + r \cdot u' \cdot dx = \ell \cdot ds + s \cdot \ell' \cdot dx$$

u is equal to ℓ at all intersection points so we can divide the left side by u and the right side by ℓ. r and s drop out because they are equal to 1.

$$dr + \frac{u'}{u} dx = ds + \frac{\ell'}{\ell} dx$$

$$dr - ds = \frac{\ell'}{\ell} dx - \frac{u'}{u} dx$$

By multiplying the right side by x/x the solution can be given in terms of the elasticities.

$$dr - ds = \ell' \frac{x}{\ell} \frac{dx}{x} - u' \frac{x}{u} \frac{dx}{x}$$

$$dr - ds = (E_{\ell d} + E_u) \frac{dx}{x}$$

where $E_u = - \frac{du}{dx} \frac{x}{u}$.

$$\frac{dx}{x} = \frac{dr - ds}{E_{\ell d} + E_u}$$

Substituting the terms appropriate for a 1 percent wage change we find:

$$\text{elasticity of supply} = \frac{dx}{x} = \frac{1 - E_u - \ell d^*}{E_{\ell d} + E_u}$$

There are three interesting properties of this formula.

(1) $E_s > -1$

(2) The sign of the slope is determined by the relation: $E_u + \ell d^* : 1$

(3) The numerical value of the slope is determined by the two quantities: $E_u + E_{\ell d}$ and $E_u + \ell d^*$

The first two properties are, more or less, intuitively obvious. If the wage rate increased 1 percent, we would not expect the labor supply to decrease more than 1 percent. In the second case, using a wage increase as example, if there was a large change in the marginal utility of leisure and a large change in the marginal utility of income, we would be led to expect a decrease in supply and consequently a negative supply curve. The third property is more subtle in that we would be more apt to think in terms of the ratio of the values. A close analysis of the graph will also reveal that it is the sum that is important, however.

It is quite possible that ℓd^* is so small that it can be neglected. In other words, a wage change will have little effect on the total disutility of work. This is most apt to be true if there is a considerable amount of leisure (45 hour work week or less). The elasticity of supply in this case becomes $\frac{1 - E_u}{E_u + E_d}$. In this situation a negative supply curve can only result if the marginal utility of money falls more than 1 percent for a 1 percent rise in income. The slopes of the leisure and direct disutility of work curves are important in determining the numerical value of E_s, but do not determine the sign of the slope.

With our present knowledge we cannot attach numerical values to these elasticities. However, it is possible to gain more insight into the two questions posed at the beginning of this section with the aid of the above formulas.

This author can see no logical reason for believing that the supply curve changes from positive to negative slope as the wage increases. It is possible that the supply curve is negative, but it does not necessarily follow that it is more apt to be negative at any particular point. To make a statement about the change in slope due to a wage change requires a knowledge of the third derivative of the utility function. The standard error of an estimate of this quantity is undoubtedly large.

The second question dealt with the hypothesis that the supply curve varies considerably depending on the type of individual. This aspect will now be investigated by analyzing the supply similarly for the following groups: (1)

men who are head of a household, (2) members of a family unit not head of a household, (3) single persons not members of a family unit. A family unit is a situation where each member usually contributes his wages to the benefit of the whole family. This is most evident in the case where the wife works to supplement the family income.

Figures 1, 2 and 3 represent the three different groups. Figure 4 is representative of an individual who does not desire to work at the wage W_0. The $\ell d \ell d$ curve lies above uu at all points. If there is a restriction on the minimum hours per week, the diagram can be extended to consider this situation by drawing a vertical line as shown at 20. The labor force status will then depend on the relation between the area under the uu curve and the area under the $\ell d \ell d$ curve between zero and the minimum hours per week.

The first group comprises the largest portion of the labor force. It will be useful to consider Figure 1 as a standard or norm. The uu curve for the second group bears an interesting relationship to the uu curve in Figure 1. The curve in Figure 2 will be the portion of the uu curve in Figure 1 that lies to the right of z. This is because the income of these individuals is a supplement to the income of the head of the household.

The $\ell\ell$ and dd curves will be expected to be quite different from those shown in group one. Members of this group have other activities such as household chores and child care. The $\ell\ell$ curve will include the value of time for these activities and will therefore, likely be higher than that shown in Figure 1. The annoyance of work will also differ because of the difference in physical stamina of the two groups. The most obvious difference between the two groups is that the situation shown in Figure 4 is more apt to hold true for the groups represented by Figure 2 than for the groups represented by Figure 1.

In the third group (Figure 3) the uu represents the marginal utility of money to the worker himself, while in the first group it represented the marginal utility of income to a family unit. The marginal utility of leisure may also be quite different. As it shows, the marginal utility for a large amount of leisure is probably higher in the case of single persons. School and organizational activities would also give a high value to leisure hours.

No definite statements can be made about the variation in supply functions for these various groups. The above analysis did show that the component parts are likely to vary considerably between the groups. Consequently, there is no reason to assume that the supply functions are similar. No particular merit is claimed for this three-fold grouping. The purpose was only to show that the supply functions for various individuals are likely to be considerably different.

This theoretical analysis further illustrates the need for empirical research. Hypotheses can be made, but no confidence or reliability can be attached without analyzing actual data.

From the analysis it seems that the two hypotheses considered in this chapter are worthy of further discussion and investigation. The analysis pointed out that there is really no reason to assume that the supply function should be the same for different groups of individuals. It, therefore, seems best to obtain data by sex and age so that we can investigate this aspect further rather than continuing to derive functions from aggregate data.

Very little can be said concerning the possible change in the supply function as the wage increases. There is considerable emphasis on this aspect at the present, and only empirical investigation can settle the issue.

Need for More Pertinent Data

Lack of pertinent data is a major obstacle in wage theory. Information is incomplete, subject to considerable error in measurement, and in many cases, only available for a short span of years. A time series study is hindered by the lack of consistent series on most aspects.

Effort should be directed along three lines: We should strive to find out what data should be gathered; data should be collected in a manner that will produce a consistent series for many years; and much effort should be spent improving accuracy and efficiency of collection. The latter two are to a certain extent incompatible in that an improvement may be such that the improved series may not be comparable to the earlier series. This makes it most important to use the best possible procedure at the beginning of a series.

A sample survey makes it possible to obtain data on designated areas, either geographical or population. Current data in the population report series is quite successfully being collected through sampling. Data should be collected on the basis of age, sex, and family status in an effort to determine if the supply functions vary between various groups.

Fringe benefits are becoming an increasingly important element in costs and benefits. Methods for measuring the extent of these benefits must be developed so that some measure of the effect they exert on the individuals concerned can be taken into consideration.

The particular data to be collected should depend on the nature of the subject, questions to be answered, techniques of analysis to be used, and of course, economy in collecting. It is not feasible to think in terms of "all-purpose" data. The purposes for which the data are to be used are too diverse and the methods of collecting are inadequate (1).

There is a large amount of data being collected that does not readily lend itself to theoretical analysis (49). A partial explanation might well be that there is no close connection between those engaged in gathering wage information and those who have occasion to use it (35). It is especially important that government agencies receive more guidance from analysts as to the type of data to be collected (28).

Information on employment and unemployment is some of the most difficult data to collect (31). Many problems will arise in collecting the type of data that the theorists need. The definition and specification of data series may differ depending upon the need and questions of the research worker (16). Obtaining data for special purposes will increase the quantity of data several fold. However, it is unreasonable to hope for substantial improvement in theory if theoretical analysis continues to be a by-product in the use of economic statistics.

Methodology

The results from attempts in empirical work, such as those presented in this paper and those by Paul Douglas, are quite encouraging. It appears to the writer that it is better to orient future work toward empiricism rather than to contribute more untested theories. All research points to the fact that this aspect of economics is extremely complex. In such a situation it is very easy for theories and knowledge to become vague and confused. To alleviate this condition we must orient our work in terms of significant and testable hypotheses.

There are so many factors that are likely to have influence on labor supply that it seems nearly impossible that we can develop a theory incorporating all of them. Our first efforts in the new approach should be to determine which of these possible factors are most important and then endeavor to obtain a quantitative measure of their importance. It is always possible that the information obtained from an analysis of specific questions can afterwards be integrated into a more general theory.

It would be desirable to establish some sort of hierarchy of questions or factors to investigate. This hierarchy should be based on the importance of the questions and the availability of methods adequate for answering them. We must disregard professional prejudices and establish this list in the light of supporting evidence. We know that the economic aspect is important, but it may well be that noneconomic aspects are more important in some instances. At least for the present we should continue to build economic models and treat important noneconomic factors as special cases. If further empirical research indicates that economic factors are not the most important then we should consider abandoning the economic model as the basic model for wage theory. Institutional factors are already receiving considerable attention (36).

Data at the present time is very poor in quality and quantity. Part of the reason for such poor data is that it has not been collected for research purposes. Much of the data has been collected by government agencies as sort of an "all purpose" data. In many cases it seems that very little use is made of the massive amount of data collected by the various government bureaus. This indicates that data should be collected for more specific purposes.

Some factors can only be studied in time series analysis. In these cases it is essential that we start collecting data as soon as possible so that data for successive years are available. The method of collecting should be such that it will be applicable for several years, thereby eliminating the obstacles of a discontinuous series so prevalent in the present data series. At the same time we should experiment to determine best methods of collecting and to determine which data are valuable. This long range collecting program and short range experimenting will have to be integrated.

Research effort should be guided by three factors: the question to be answered, the data available, and the techniques of analysis that are applicable. Disregard for any one of these will seriously limit progress at times. We may have to be satisfied with mediocre results; this is to be preferred to absence of effort excused by adverse odds.

We cannot expect that an answer to a specific question will hold true for all time. The same questions will likely have to be reanswered as time brings new changes. Even the best method of answering these questions will change. There is no one "scientific" method that is best for all questions, nor one that is best for answering one specific question at all times. This being the case, all studies are also valuable as studies in methodology. Methods cannot be tested and improved unless they are applied many times.

To bridge the gap between theoretical concepts and the real world, it is necessary to have a procedure of identification. Different branches of applied economics should provide such procedures. Just as important is a procedure of verification (testing). This is a complex aspect because theoretical economics are never borne out exactly by empirical observation. More difficulty is encountered in this step because the assumptions underlying the statistical methods are not entirely fulfilled. Rules for judging which hypotheses to accept as empirically verified or to reject as empirically unverified are, as yet, arbitrary. Econometrics is a branch of economics that deals with the procedures of verification. It is hoped that this work is a worthy application of this methodology.

LITERATURE CITED

1. Bancroft, G. and H. Welch. Recent experience with problems of labor force measurements. American Statistical Association J. 41: 303-312. 1946.

2. Benjamin, A. C. Operationism. Springfield, Illinois, Charles C. Thomas. 1955.

3. Bituminous Coal Institute. Bituminous Coal Annual 1953. Washington, D. C., The Institute. 1953.

4. Boulding, K. In defense of statics. Quarterly J. of Economics. 69: 485-502. 1955.

5. Break, G. F. Income taxes, wage rates, and the incentive to supply labor services. National Tax J. 6: 333-352. 1953.

6. Clark, C. Estimates of average hours worked per week. Review of Economic Progress. 3: 78-84. 1951.

7. Cohen, M. R. Generalization in the social sciences. In Wirth, L., ed. Eleven twenty-six. Chicago, Illinois, The Univ. of Chicago Press. 1940.

8. Collingwood, A. G. Economics as a philosophical science. International J. of Ethics. 36: 162-185. 1926.

9. Davey, H. Contemporary collective bargaining. New York, New York, Prentice-Hall, Inc. 1951.

10. Douglas, P. American wage determination: the trend and its significance. Washington, D. C., U. S. Chamber of Commerce. 1947.

11. ________ Are there laws of production? American Economic Review. 38: 1-41. 1948.

12. ________ Real wages in the United States. Cambridge, Massachusetts, Houghton Mifflin Co. 1930.

13. ________ Theory of wages. New York, New York, Macmillan Co. 1934.

14. Dunlop, J. T. The task of contemporary wage theory. In Taylor, G. W. and F. C. Pierson, eds. New concepts in wage determination. pp. 117-139. New York, New York, McGraw-Hill Book Co., Inc. 1957.

15. ________ Wage determination under trade unions. New York, New York, Macmillian Co. 1944.

16. Economic Statistics Bureau of Washington, D. C. Handbook of basic economic statistics. Washington, D. C., The Bureau. Jan. 1957.

17. Federal Reserve Bank of New York. Composite index of wages and salaries. New York, New York, Federal Reserve Bank of New York. Aug. 1955. (Mimeo. report.)

18. ________ Index of estimated composite wages in U. S. New York, New York, Federal Reserve Bank of New York. 1938 revision. (Mimeo. report.)

19. ________ Index of wages. New York, New York, Federal Reserve Bank of New York. 1939 revision. (Mimeo. report.)

20. Hempel, C. G. Fundamentals of concept formation in empirical science. International Encyclopedia of Unified Science. 2, no. 7: 39-50. 1952.

21. Hicks, J. R. The theory of wages. London, Macmillan and Co., Limited. 1932.

22. Kerr, C. Labor markets: their character and consequences. Industrial Relations Associations Proc. 1949, part 1: 69-99. 1949.

23. Keynes, J. M. The general theory of employment interest and money. New York, New York, Harcourt, Brace and Co. 1935.

24. ________ Professor Tinbergen's method. Economic J. 49: 558-568. 1939.

25. Lange, O. The scope and method of economics. In Feigl, H. and M. Brodbeck, eds. Readings in the philosophy of science. pp. 744-754. New York, New York, Appleton-Century-Crofts, Inc. 1953.

26. Mills, F. C., F. H. Knight, O. Lange, J. Marschak, J. Mayer, S. Rice, J. Viner, and T. O. Yntema. Quantification: the quest for precision. In Wirth, L., ed. Eleven twenty-six. pp. 153-193. Chicago, Illinois, The Univ. of Chicago Press. 1940.

27. Modigliani, F. Liquidity preference, interest and money. Econometrica. 12: 45-88. 1944.

28. More, G. Accuracy of government statistics. Harvard Business Rev. 25: 306-317. 1947.

29. Morgenstern, O. On the accuracy of economic observations. Princeton, N. J., Princeton Univ. Press. 1950.

30. Myers, C. A. Empirical research on wages. Industrial Relations Research Association Proc. 1949, part 8: 234-240. 1949.

31. National Bureau of Economic Research. The measurement and behavior of unemployment. Princeton, N. J., Princeton Univ. Press. 1957.

32. National Industrial Conference Board. The economic almanac. New York, New York, Thomas Y. Crowell Co. 1956.

33. Neumann, J. von and O. Morgenstern. Theory of games and economic behavior. 2nd ed. Princeton, N. J., Princeton Univ. Press. 1947.

34. Pierson, F. C. An evaluation of wage theory. In Taylor, G. W. and F. C. Pierson, eds. New concepts in wage determination. pp. 3-31. New York, New York, McGraw-Hill Book Co., Inc. 1957.

35. Raimon, R. L. Sources of wage data. Industrial Relations Research Proc. 1953, part 8: 241-251. 1953.

36. Reynolds, L. G. and C. Taft. The evolution of wage structure. New Haven, Conn., Yale Univ. Press. 1956.

37. ________ The state of wage theory. Industrial Relations Research Proc. 1953, part 8: 252-265. 1953.

38. Ross, A. M. Trade union wage policy. Berkeley, Calif., Univ. of Calif. Press. 1948.

39. Rothbaum, M. National wage-structure comparisons. In Taylor, G. W. and F. C. Pierson, eds. New concepts in wage determination. New York, New York, McGraw-Hill Book Co., Inc. 1957.

40. Rothschild, K. W. The theory of wages. New York, New York, Macmillan Co. 1954.

41. Slichter, S. H., Notes on the structure of wages. Review of Economics and Statistics. 28: 80-91. 1950.

42. Theil, H. Linear aggregation of economic relations. In Tinbergen, J., P. J. Verdoorm, and J. S. Witteveen, eds. Contributions to Economic Analysis. no. 7. Amsterdam, Holland, North-Holland Publishing Co. 1954.

43. Tintner, G. An application of the variate difference method to multiple regression. Econometrica. 12: 97-113. 1944.

44. ________ Econometrics. New York, New York, John Wiley and Sons. 1952.

45. ________ Homogeneous systems in mathematical economics. Econometrica. 16: 273-294. 1948.

46. ________ Mathematics and statistics for economists. New York, New York, Rinehart and Co., Inc. 1948.

47. ________ The use of mathematical methods in econometrics and economic statistics. International Social Science Bulletin. 6, no. 4: 1-12. 1954.

48. ________ The variate difference method. Bloomington, Indiana, Principia Press, Inc. 1940.

49. Tolles, N. A. and R. L. Raimon. Sources of wage information: employer associations. Ithaca, New York, Cornell Univ. Press. 1952.

50. U. S. Dept. of Commerce. Bureau of the Census. Current population reports. Series P-25, nos. 73, 78, 146; Series P-50, no. 63; Series P-57, no. 150. Washington, D. C., U. S. Government Printing Office. 1956.

51. ________ ________ Historical statistics of the U. S. Washington, D. C., U. S. Government Printing Office. 1949.

52. ________ ________ Statistical Abstract of the U. S. for 1956, 1957, 1958. Washington, D. C., U. S. Government Printing Office. 1956, 1957, 1958.

53. Woytinsky, W. S., T. C. Fichandler, M. C. Bishop, and M. R. Gannett. Employment and wages in the United States. Baltimore, Maryland, The Lord Baltimore Press. 1953.

54. Yoder, D. Demands for labor: opportunities for research. Social Science Research Council. no. 7: 1-40. 1948.

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APPENDICES

APPENDIX A

Table 4. Difference analysis[a]

Order of difference k	Sum of squares	$A_{n,k}$[b]	Variance of difference V_k[c]	$H_{n,k}$[d]	Standard error ratio R_K[e]	Probability
			Supply 1900-1928, n = 19			
0	.04179		.001490	5.197	1.577	.1140
1	.00572	.018149	.0001038	10.007	3.344	.0008
2	.01100	.006283	.00006911	12.882	2.017	.0434
3	.02971	.001961	.00005829	14.779	1.605	.1074
4	.08893	.0005843	.00005196	15.833	1.639	.1010
5	.27467	.0001696	.00004658	16.464	1.152	.2502
6[f]	.89810	.00004823	.00004332	16.742	-.054	.9600
7	3.17940	.00001367	.00004346			
			Wage rate 1900-1928			
0	.94111		.033610	5.197	5.116	.00003
1	.02877	.018149	.0005221	10.007	7.468	.00002
2	.02123	.006283	.0001334	12.882	3.546	.00004
3	.04930	.001961	.00009668	14.779	1.680	.0930
4	.14666	.0005843	.00008569	15.833	.296	.7642
5[f]	.49584	.0001696	.00008409	16.464	-.188	.8492
6	1.76349	.00004823	.00008505			

[a]Adapted from Tintner (48, pp. 22-72).

[b]*Ibid.*, pp. 43 ff.

[c]$V_k = SS_k A_{n,k}$

[d]Adapted from Tintner (48, pp. 57 ff.).

[e]$R_K = \frac{(V_k - V_{k+1})H_{kn}}{V_k}$

[f]The variance for this difference was taken as a measure of the error variance.

Table 4. (Continued)

Order of difference	Sum of squares	$A_{n,k}$	Variance of difference V_k	$H_{n,k}$	Standard error ratio R_B	Probability
			Consumer price index 1900-1928			
0	.58938		.021050	5.197	5.083	.00003
1	.02497	.018149	.0004532	10.007	6.317	.00002
2	.02659	.006283	.0001671	12.882	1.488	.1362
3	.07539	.001961	.0001478	14.779	6.020	.00002
4	.14990	.0005843	.00008759	15.833	2.377	.0192
5	.43323	.0001696	.00007348	16.464	1.653	.0990
6	1.37049	.00004823	.00006610	16.742	1.099	.3074
7	4.51813	.00001367	.00006176	16.833	.868	.3842
8	15.36160	.000003824	.00005874	16.659	.428	.6670
9	53.58036	.000001068	.00005723	16.286	.148	.8008
10[f]	190.29117	.000000298	.00005671			
			Supply 1929-1955, n = 16			
0	.015265		.000611	4.884	2.950	.0032
1	.011580	.020872	.000242	9.300	3.534	.0004
2	.020620	.007275	.000150	11.845	.213	.8336
3[f]	.057620	.0022886	.000132	13.390	0.000	1.0000
4	.192050	.0006870	.000132	14.277	-.756	.4472
5	.690380	.00008020	.000139			
			Wage rate 1929-1955			
0	.23064		.009230	4.884	2.446	.0142
1	.022100	.020872	.000461	9.300	7.180	.00002
2	.014460	.007275	.000105	11.845	3.648	.0002
3	.031770	.002288	.0000726	13.390	2.999	.0028
4	.081961	.0006870	.0000563	14.277	3.212	.0014
5	.216980	.0002010	.0000436	14.714	3.237	.0012
6	.568120	.0000580	.0000340	14.835	2.522	.0138
7	1.710910	.0000165	.0000282	14.730	.930	.3524
8[f]	5.201120	.000004706	.0000245			

[f]The variance for this difference was taken as a measure of the error variance.

Table 4. (Continued)

Order of differ-ence	Sum of squares	$A_{n,k}$	Variance of differ-ence V_k	$H_{n,k}$	Stand-ard error ratio R_B	Proba-bility
			Consumer price index 1929-1955			
0	.122299		.004892	4.884	4.557	.00004
1	.015621	.020872	.000326	9.300	7.421	.00002
2	.009040	.007275	.0000658	11.845	5.088	.00003
3	.017840	.002288	.0000408	13.390	3.983	.00006
4	.042790	.0006870	.0000294	14.277	5.054	.00003
5	.094460	.0002010	.0000190	14.714	2.781	.0056
6	.265160	.0000580	.0000154	14.835	1.445	.1470
7	.844090	.0000165	.00001391	14.730	.212	.8336
8[f]	2.900750	.000004706	.0000137			
			Demand for labor, bituminous coal 1929-1955, n = 17			
0	.167797		.0104873	4.9884	4.909	.00003
1	.083450	.019964	.001666	9.5358	5.020	.00003
2	.113590	.006944	.0007888	12.1906	1.762	.0786
3	.309670	.002179	.0006748	13.8297	1.535	.1236
4	.918870	.006529	.0005999	14.7956	.937	.3472
5	2.944870	.0001908	.005619	15.2971	.539	.5892
6	9.899390	.00005476	.0005421	15.4709	.271	.7872
7[f]	34.118120	.00001561	.0005326	15.4080	-.043	.9680
			Wage rate, bituminous coal 1929-1955			
0	.799581		.0499738	4.9884	4.887	.00003
1	.050900	.019964	.0010162	9.5358	5.940	.00002
2	.055190	.006944	.0003832	12.1906	3.079	.0020
3	.131450	.002179	.0002864	13.8297	1.954	.0512
4	.376710	.006529	.0002460	14.7956	1.810	.0702
5	1.131450	.0001908	.0002159	15.2971	1.459	.1442
6	3.566800	.00005476	.0001953	15.4709	.475	.6312
7	12.127870	.00001561	.0001893	15.4080	.252	.8026
8[f]	42.012780	.000004432	.0001862	15.1729	.824	.4122
9	140.965790	.000001246	.0001756			

[f]The variance for this difference was taken as a measure of the error variance.

Table 4. (Continued)

Order of difference	Sum of squares	$A_{n,k}$	Variance of difference V_k	$H_{n,k}$	Standard error ratio R_B	Probability
	Wholesale price index, bituminous coal 1929-1955					
0	.685250		.0428281	4.9884	4.618	.00003
1	.049730	.019964	.0009928	9.5358	5.940	.00002
2	.053920	.006944	.0003744	12.1906	3.455	.0006
3	.123130	.002179	.0002683	13.8297	2.866	.0040
4	.325760	.0006529	.0002127	14.7956	2.678	.0074
5	.912811	.0001908	.0001742	15.2971	2.529	.0132
6	2.654790	.00005476	.0001454	15.4709	1.747	.0802
7	8.266790	.00001561	.0001290	15.4080	1.373	.1706
8	26.510360	.000004432	.0001175	15.1729	2.027	.0424
9	81.703420	.000001246	.0001018	14.8116	1.481	.1388
10[f]	237.791030	.000000352	.0000837			

[f]The variance for this difference was taken as a measure of the error variance.

APPENDIX B

Table 5. Data analyzed for labor supply

Year	Employed (1,000)[a]	Unemployed (1,000)[b]	Population (15-64)[c]	Supply (%)[d]	Av. hours worked per week[e]	Index of average hourly earnings (1939 = 100)[f]	Consumer price index (1935-1939 = 100)[g]	Retail prices[h]	War years[i]
1900	27,378	1,779	46,780	91.0	58.4	28.3	54.696		0
01	28,238	1,331	47,830	89.2	57.7	29.1	55.728		0
02	30,405	1,300	48,969	92.4	57.1	30.3	57.276		0
03	30,319	1,329	50,052	89.5	56.6	31.6	59.856		0
04	31,175	2,025	51,187	91.0	56.1	31.9	59.340		0
05	33,032	1,412	52,417	91.7	55.8	32.4	59.340		0
06	34,790	1,299	53,622	92.9	55.2	33.7	61.404		0
07	34,875	1,529	54,796	91.0	54.8	34.8	65.016		0
08	34,284	3,767	56,054	92.0	54.2	34.6	62.436		0
09	36,735	2,097	57,355	91.1	53.8	35.0	62.436		0

[a]See Ref. (9, p. 65), (52, 1947, p. 220), (51, p. 227), (52, 1956, p. 197). The figures represent total employed minus the number in active military service. For a description of the revision of the data series after 1952 see Ref. (50, Series P-57).

[b]See Ref. (2, p. 460), (51, p. 65), (51, p. 197). Estimates for the years 1900-1926 were multiplied by the factor .606 to make them comparable to the estimates by Leo Wolman (51, p. 427) for the years 1920-1926. Estimates for 1927 and 1928 were made by linear interpolation between 1926 and 1929.

[c]See Ref. (51, p. 26), (52, 1947, p. 220), (50, Series P-25, nos. 73, 78), (50, Series P-25, no. 146), (52, 1956, p. 235). The figures represent total population between 15 and 64 years of age minus the number in active military service.

(footnotes continued on next page)

Table 5. (Continued)

Year	Employed (1,000)	Unemployed (1,000)	Population (15-64)	Supply (%)[d]	Av. hours worked per week[e]	Index of average hourly earnings (1939 = 100)[f]	Consumer price index (1935-1939 = 100)[g]	Retail prices[h]	War years
1910	37,580	1,730	58,721	89.2	53.3	35.7	66.048		0
11	37,097	2,242	59,702	87.0	52.8	36.3	68.112		0
12	38,169	1,675	60,653	85.9	52.3	37.4	68.628		0
13	38,482	2,025	61,869	84.6	51.7	38.8	70.7		0
14	37,575	4,129	63,057	84.2	50.9	39.2	71.8		0
15	37,728	3,914	63,936	82.7	50.8	39.6	72.5		0
16	40,127	1,585	64,802	81.3	50.5	43.2	77.9		0
17	42,685	1,594	65,554	84.6	50.1	48.9	91.6		1
18	44,187	1,508	64,144	88.0	49.4	59.8	107.5		1
19	42,029	1,843	65,957	80.8	48.6	70.3	123.8		0

[d]Labor supply is expressed as a percentage and was obtained by the following calculation:

$$S = \frac{(\text{no. employed} + \text{no. unemployed}) \times (\text{av. hrs. worked per week})}{(\text{pop. 15-64}) \times (\text{40 hrs. per week})}$$

The denominator is a measure of the potential labor supply with the 40 being an arbitrary figure.

[e]See Ref. (6, pp. 78-84), (50, Series P-50).

[f]See Ref. (52, 1956, p. 295), (18), (19), (17). The average hourly earnings includes wage supplements.

[g]See Ref. (51, p. 233), (52, 1956, p. 234).

[h]See Ref. (32, p. 75). Data for years prior to 1941 are not available.

(footnotes continued on next page)

Table 5. (Continued)

Year	Employed (1,000)	Unemployed (1,000)	Population (15-64)	Supply (%)	Av. hours worked per week	Index of average hourly earnings (1939 = 100)	Consumer price index (1935-1939 = 100)	Retail prices	War years[1]
1920	41,339	1,903	67,546	75.5	47.2	85.2	143.3		0
21	37,691	5,136	68,841	71.9	45.2	78.6	127.7		0
22	40,049	5,000	69,858	77.0	47.8	75.6	119.7		0
23	43,011	2,169	71,197	75.7	47.7	81.4	121.9		0
24	42,515	3,547	72,737	72.8	46.0	84.1	122.2		0
25	44,192	2,523	73,865	74.2	46.9	84.6	125.4		0
26	45,498	2,144	75,093	74.1	46.7	86.0	126.4		0
27	45,319	2,893	76,395	73.2	46.4	87.4	124.0		0
28	46,067	2,940	77,658	73.3	46.5	88.1	122.6		0
29	47,630	1,550	78,835	73.0	46.8	90.0	122.5		0

[1]A dummy variable was included which was given a value of one for war years and a value of zero in all other years. The war years were first designated on the basis of the size of the armed forces and the change from the previous year. In 1917 the armed forces increased considerably from the previous year and remained high through 1918. In 1941 there was again a considerable increase over the previous year and the number in the active forces remained high through 1945. In 1946 there was a considerable decrease. No war period was designated for the Korean episode because universal military training kept the armed forces about the same size for several years. The influences of the police action extended quite uniformly over several years. Upon further analysis of the data it was found that in the years 1943-1946 the war seemed to have a much greater effect than in the years 1942-1943. For studying the supply of labor it seemed best to designate the war years as 1917-1918 and 1943-1945.

Table 5. (Continued)

Year	Employed (1,000)	Unemployed (1,000)	Population (15-64)	Supply (%)	Av. hours worked per week	Index of average hourly earnings (1939 = 100)	Consumer price index (1935-1939 = 100)	Retail prices	War years
1930	45,480	4,340	80,112	72.8	44.8	89.8	119.4		0
31	42,400	8,020	81,050	67.8	43.6	86.9	108.7		0
32	38,940	12,060	81,932	63.5	40.8	77.2	97.6		0
33	38,760	12,830	82,829	64.9	41.7	75.1	92.4		0
34	40,890	11,340	83,811	62.9	40.5	83.6	95.7		0
35	42,260	10,610	84,812	64.8	41.6	86.0	98.1		0
36	44,410	9,030	85,749	67.1	43.1	87.7	99.1		0
37	46,300	7,700	86,683	65.4	42.0	97.0	102.7		0
38	44,220	10,390	87,672	59.2	38.0	100.0	100.8		0
39	45,750	9,480	88,662	62.9	40.4	100.0	99.4		0
1940	47,520	8,120	89,747	63.5	41.0	102.0	100.2		0
41	50,350	5,560	89,368	66.6	42.6	107.0	105.2	108.3	0
42	53,750	2,660	88,235	70.3	44.0	122.0	116.6	124.9	0
43	54,470	1,070	83,917	80.3	48.5	133.0	123.7	134.0	1
44	53,960	670	82,466	77.2	46.6	136.0	125.7	139.5	1
45	52,820	1,040	82,502	75.2	46.1	145.0	128.6	141.4	1
46	55,250	2,270	92,469	68.9	44.3	162.0	139.5	155.2	0
47	58,027	2,141	94,717	69.1	43.5	178.0	159.6	180.1	0
48	59,378	2,064	95,623	68.8	42.8	191.0	171.9	192.7	0
49	58,710	3,395	96,281	67.9	42.1	202.0	170.2	187.7	0

Table 5. (Continued)

Year	Employed (1,000)	Unemployed (1,000)	Population (15-64)	Supply (%)	Av. hours worked per week	Index of average hourly earnings (1939 = 100)	Consumer price index (1935-1939 = 100)	Retail prices	War years
1960	59,967	3,142	97,076	69.1	42.5	211.0	171.2	189.0	0
61	61,005	1,879	96,075	69.7	42.6	231.0	185.6	206.9	0
62	61,293	1,703	96,491	69.34	42.5	244.0	189.8	210.4	0
63	61,838	1,602	97,269	68.65	42.1	256.0	192.7	208.5	0
64	60,863	3,230	98,308	67.80	41.6	264.0	193.3	209.1	0
65	62,859	2,654	99,438	69.01	41.9	272.0	192.8	208.6	0

APPENDIX C

Table 6. Data analyzed for labor demand in bituminous coal industry

Year	No. employed[a]	Av. hrs. worked per wk.[b]	Demand (million hrs.)[c]	Av. hourly earnings[d]	Wholesale price index (1926 = 100)[e]	War years
1900	304,375	33.3	10.14	.202	32.3	0
01	340,235	33.2	11.30	.229	32.6	0
02	370,056	33.1	12.25	.242	34.7	0
03	415,777	33.1	13.76	.264	38.5	0
04	437,832	32.7	14.32	.268	34.1	0
05	460,629	32.7	15.06	.273	32.9	0
06	478,425	32.7	15.64	.290	34.4	0
07	513,258	32.7	16.78	.285	35.4	0
08	516,264	32.7	16.88	.290	34.7	0
09	543,152	32.7	17.76	.289	33.2	0
1910	555,533	32.7	18.17	.296	34.7	0
11	549,775	32.7	17.98	.302	34.4	0
12	548,632	32.7	17.94	.317	35.7	0
13	571,882	32.7	18.70	.313	38.1	0
14	583,506	32.7	19.08	.320	34.8	0
15	557,456	32.7	18.23	.334	33.9	0
16	561,102	32.7	18.35	.375	55.5	0
17	603,143	31.5	19.00	.479	98.4	1
18	615,305	30.8	18.95	.595	81.4	1
19	621,998	30.7	19.10	.692	79.8	0

[a]See Ref. (3, p. 24), (16, p. 54).

[b]See Ref. (52, 1956, p. 150), (51, p. 68), (16, p. 54).

[c]Demand is measured as the total number employed multiplied by the average hours worked per week.

[d]See Ref. (12, p. 152), (51, p. 68), (16, p. 54).

[e]See Ref. (3, p. 24), (16, pp. 118, 127).

Table 6. (Continued)

Year	No. employed	Av. hrs. worked per wk.	Demand (million hrs.)	Av. hourly earnings	Wholesale price index (1926 = 100)	War years
1920	639,547	30.5	19.51	.776	165.4	0
21	663,754	30.5	20.24	.837	77.7	0
22	687,956	31.3	21.53	.825	113.1	0
23	704,793	31.3	22.06	.845	113.4	0
24	619,604	30.0	18.59	.813	99.7	0
25	588,493	34.2	20.13	.800	96.5	0
26	593,647	37.7	22.38	.786	100.0	0
27	593,918	33.5	19.90	.751	100.3	0
28	522,150	35.6	18.59	.716	93.6	0
29	502,993	38.4	19.31	.681	91.3	0
1930	493,202	33.5	16.52	.684	89.4	0
31	450,213	28.3	12.74	.647	84.6	0
32	406,380	27.2	11.05	.520	82.0	0
33	418,703	29.5	12.35	.501	82.8	0
34	458,011	27.0	12.37	.673	94.5	0
35	462,403	26.4	12.21	.745	96.7	0
36	477,204	28.8	13.74	.794	97.4	0
37	491,864	27.9	13.72	.856	98.6	0
38	441,333	23.5	10.37	.878	99.0	0
39	421,788	27.1	11.43	.886	97.5	0
1940	439,075	28.1	12.34	.883	97.6	0
41	456,981	31.1	14.21	.993	104.3	1
42	461,991	32.9	15.20	1,059	109.7	1
43	416,007	36.6	15.23	1.139	116.1	1
44	393,347	43.4	17.07	1.186	120.3	1
45	383,100	42.3	16.21	1.240	123.1	1
46	396,434	41.6	16.49	1.481	132.5	0
47	419,182	40.7	17.06	1.636	157.6	0
48	441,631	38.0	16.76	1.898	187.0	0
49	433,698	32.6	14.14	1.941	191.9	0

Table 6. (Continued)

Year	No. employed	Av. hrs. worked per wk.	Demand (million hrs.)	Av. hourly earnings	Wholesale price index (1926 = 100)	War years
1950	415,582	35.0	14.55	2.010	193.7	0
51	372,297	35.2	13.13	2.210	196.2	0
52	335,217	34.1	11.43	2.290	196.7	0
53	295,500	34.4	10.17	2.480	204.2	0
54	231,900	32.6	7.56	2.480	192.4	0
55	219,922	30.7	7.17	2.560	189.7	0